



Geometry of a Chaos: Advanced computational approach to treating chaotic dynamics of environmental radioactivity systems II

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Abstract In the paper we go on our work on application of a chaos geometry tools and non-linear analysis technique to studying chaotic features of different nature systems. Here there are presented the results of using an advanced chaos-geometric approach to treating chaotic dynamics of environmental radioactivity systems. A usually, an approach combines together application of the advanced mutual information scheme, Grassberger-Procachi algorithm, Lyapunov exponent's analysis etc.

Keywords Geometry of chaos, non-linear analysis, radioactivity systems

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In this paper we go on our work on application of the chaos theory and non-linear analysis technique to studying chaotic features of different nature systems (see, for example [1-7]). The theoretical basis's of the chaos-geometric combined approach to treating of chaotic behaviour of complex dynamical systems are in details in series of ref. [1-10]. Generally, an approach combines together application of the advanced mutual information scheme, Grassberger-Procachi algorithm, Lyapunov exponent's analysis etc. It is important to note that this approach has been successfully applied to studying dynamics not only mathematical and

physical systems. Very impressive application is the investigated dynamics of the atmospheric pollutants concentrations and forecasting their temporal evolution. Besides, in Ref [2] it has been numerically vstudied a chaotic dynamics of the pollutants concentration in some hydroecological, namely, water system. The successful application of new chaos-geometrical approach to studying dynamics of the different nature systems demonstrates its universal character.

Here, starting from our previous works (see, for example [1-7]), we present the possibilities of using a chaos-geometric approach to treating chaotic dynamics of environmental radioactivity systems. In fact, speech is about a perspective application of a chaos geometry tools to treating very important applied problem. Let us remind that hitherto the different mathematical modeling methods deal with serious principal and numerical difficulties under studying the key radioecological transfer and effects such as radionuclide cycles in the ecosystems, radionuclide transfer to biota in inland waters, biological effects of radiation exposure to living organisms etc.

As usually, following to [1-10], we formally consider scalar measurements of some fundamental characteristics radioactivity systems dynamics (such as concentration etc) $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is a start time, Δt is time step, and n is number of the measurements. In a general case [6-7], $s(n)$ is any time series (f.e. atmospheric pollutants concentration). As processes resulting in a chaotic behaviour are fundamentally multivariate, one needs to reconstruct phase space using as well as possible information contained in $s(n)$. Such reconstruction results in set of d -dimensional vectors $\mathbf{y}(n)$ replacing scalar measurements. The main idea is that direct use of lagged variables $s(n + \tau)$, where τ is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a collection of time lags to create a vector in d dimensions, $\mathbf{y}(n) = [s(n), s(n + \tau), s(n + 2\tau), \dots, s(n + (d - 1)\tau)]$, the required coordinates are provided. In a nonlinear system, $s(n + j\tau)$ are some unknown nonlinear combination of the actual physical variables. The dimension d is the embedding dimension, d_E .

Let us remind that following to [2,10], the choice of proper time lag is important for the subsequent reconstruction of phase space. If τ is chosen too small, then the coordinates $s(n + j\tau)$, $s(n + (j + 1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. If τ is too large, then $s(n + j\tau)$, $s(n + (j + 1)\tau)$ are completely independent of each other in a statistical sense. If τ is too small or too large, then the correlation dimension of attractor can be under-or overestimated. One needs to choose some intermediate position

between above cases. First approach is to compute the linear autocorrelation function $C_L(\delta)$ and to look for that time lag where $C_L(\delta)$ first passes through 0. This gives a good hint of choice for τ at that $s(n + j\tau)$ and $s(n + (j + 1)\tau)$ are linearly independent. It's better to use approach with a nonlinear concept of independence, e.g. an average mutual information. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to 0 if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$. In ref. [4] it is suggested, as a prescription, that it is necessary to choose that τ where the first minimum of $I(\tau)$ occurs.

In [6,10] it has been stated that an aim of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The embedding dimension, d_E , must be greater, or at least equal, than a dimension of attractor, d_A , i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E , e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. If the time series is characterized by an attractor, then correlation integral $C(r)$ is related to a radius r as $d = \lim_{r \rightarrow 0, N \rightarrow \infty} \frac{\log C(r)}{\log r}$, where d is correlation exponent.

The fundamental problem of theory of radioactivity systems is in predicting its evolutionary dynamics. According to [6,7,10] the cited predictability can be estimated by the Kolmogorov entropy, which is proportional to a sum of positive LE. As usually, the spectrum of LE is one of dynamical invariants for non-linear system with chaotic behaviour. The limited predictability of the chaos is quantified by the local and global LE, which can be determined from measurements. The LE are related to the eigenvalues of the linearized dynamics across the attractor. Negative values show stable behaviour while positive values show local unstable behaviour. For chaotic systems, being both stable and unstable, LE indicate the complexity of the dynamics. The largest positive value determines some average prediction limit. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and hence the choice of trajectory, and they do comprise an invariant measure of the attractor. An estimate of this measure is a sum of the positive LE. The estimate of the attractor dimension is provided by the conjecture d_L and the LE are taken in descending order. The dimension d_L gives values close to the dimension estimates discussed earlier and is preferable when estimating high dimensions. To compute LE, we use a

method with linear fitted map, although the maps with higher order polynomials can be used too. Non-linear model of chaotic processes is based on the concept of compact geometric attractor on which observations evolve. Since an orbit is continually folded back on itself by dissipative forces and the non-linear part of dynamics, some orbit points [1,10] $\mathbf{y}^r(k)$, $r = 1, 2, \dots, N_B$ can be found in the neighbourhood of any orbit point $\mathbf{y}(k)$, at that the points $\mathbf{y}^r(k)$ arrive in the neighbourhood of $\mathbf{y}(k)$ at quite different times than k . One can then choose some interpolation functions, which account for whole neighbourhoods of phase space and how they evolve from near $\mathbf{y}(k)$ to whole set of points near $\mathbf{y}(k+1)$. The implementation of this concept is to build parameterized non-linear functions $\mathbf{F}(\mathbf{x}, \mathbf{a})$ which take $\mathbf{y}(k)$ into $\mathbf{y}(k+1) = \mathbf{F}(\mathbf{y}(k), \mathbf{a})$ and use various criteria to determine parameters $\mathbf{a}$. Since one has the notion of local neighbourhoods, one can build up one's model of the process neighbourhood by neighbourhood and, by piecing together these local models, produce a global non-linear model that capture much of the structure in an attractor itself.

3. The numerical results and conclusions

Here we briefly present the results of preliminary numerical analysis for array of values of radon flux from the Earth's surface, calculated from the experimental data of time series of radon fields (look [11] and refs. therein). Table 1 shows the correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for the four radon fields parameters series (the radon monitoring station net on the Petropavlovsk-Kamchatsky geodynamical polygon, PKGP)

Table 1. The Time lag (τ), correlation dimension (d_2), embedding dimension (d_E), Kaplan-Yorke dimension (d_L), and average limit of predictability ($Pr_{max}, hours$) for radon flux series in the PKGP.

	Serie 1	Serie 2	Serie 3	Serie 4
τ	16	16	9	7
(d_2)	4.86	4.04	3.75	4.78
(d_E)	5	5	4	5
d_L	4.98	4.64	3.28	4.36
Pr_{max}	12	10	6	8

As usually, the sum of the positive LE determines the Kolmogorov entropy, which is inversely proportional to the limit of predictability (Pr_{max}). Let us remind [6,7] since the conversion rate of the sphere into an ellipsoid along different axes is determined by the LE, it is clear that the smaller the amount of positive dimensions, the more stable is a dynamic system. Consequently, it increases the

predictability of it. As the numerical calculation shows the presence of the two (from six) positive λ_i suggests the system broadens in the line of two axes and converges along four axes that in the six-dimensional space. The time series of concentrations at the site of the Blatina watershed have the highest predictability than other time series.

Therefore, here we presented the results of an application of a chaos-geometric approach to treating of non-linear dynamics of of radon flux from the Earth's surface, calculated from the experimental data of time series of radon fields.

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