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Spaces of constant almost holomorphic curvature

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Abstract. In this work, special pseudo-Riemannian spaces endowed with an affinor structure satisfying certain naturally arising conditions are investigated. Spaces with a special form of the Riemann tensor are considered, and two types of pseudo-Riemannian spaces are distinguished depending on the type of their Riemann tensor. Their geometric characteristics are obtained for a specific form of the metric tensor. In particular, it is proven that pseudo-Riemannian spaces of constant almost holomorphic curvature with an elliptic skew-symmetric affinor structure of second degree are semi-symmetric.

The research is carried out locally, using tensor methods without imposing restrictions on the signature or definiteness of the metric tensor of the space under consideration. A special conjugation operation, introduced for spaces endowed with an affinor structure, is widely applied, along with its properties for the Riemann and Ricci tensors.

Анотація. В роботі досліджено спеціальні псевдоріманові простори, які наділено афінорною структурою, що задовольняє певним, природньо витікаючим умовам. Досліджуються простори зі спеціальним видом тензора Рімана, виділено два типи псевдоріманових просторів в залежності від типу тензора Рімана. Знайдено їх геометричні характеристики для спеціального виду метричного тензора. Зокрема доведено, що псевдоріманові простори з сталою майже голоморфною кривиною з еліптичною кососиметричною афінорною структурою другого ступеня напівсиметричні.

Дослідження ведуться локально, тензорними методами без обмежень на сигнатуру та знаковизначенність метричного тензору простору, що розглядається. В роботі широко застосовується спеціальна операція спряження, що введена для просторів, які наділено афінорною структурою, та її властивості для тензорів Рімана та Річчі.

Keywords: pseudo-Riemannian space, Riemann tensor, Ricci tensor, covariant derivative, affinor, affinor structure, semi-symmetric affinor structure, antisymmetric affinor structure, almost complex affinor structure, conjugation operation

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1. Introduction

The study of special pseudo-Riemannian spaces attracts the attention of many researchers. One of the main directions of such investigations is the specialization of spaces, that is, imposing additional conditions both on objects of internal nature and on those that are not intrinsic, meaning they do not depend on the metric tensor.

Conditions imposed on the Ricci tensor have led to the study of Einstein and almost-Einstein spaces [3, 7, 22].

Conditions on the Riemann tensor allow one to examine spaces of constant curvature, conformally flat spaces, and spaces of quasi-constant curvature, among others [2, 6, 13].

The requirement for the existence of an affinor structure turns out to be quite restrictive for many types of spaces. For example, there exist no conformally flat spaces of dimension less than four that admit a Kähler structure [24].

Spaces endowed with an affinor structure possess specific properties, both from the viewpoint of geometric characteristics and the methods used to study them [8, 9, 10].

For analogues of the conformal curvature tensor in such spaces, two types of spaces have been introduced depending on the form of the Riemann tensor. The introduction of such spaces into consideration is motivated by the necessity of specializing pseudo-Riemannian spaces. The specialization of spaces, that is, the identification of particular types of spaces, represents one of the modern directions in Riemannian geometry and facilitates the technical simplification of their study. In the present work, precisely such types have been distinguished, as they appear to be of interest from a structural viewpoint and may possess potential geometric applications. The possibility of reducing the metric tensor of these spaces to a special form is investigated.

2. On the f-structure of degree k

Consider a pseudo-Riemannian space V_n with a metric tensor $g_{ij}(x)$, in which a tensor $F_i^h(x)$ is defined. If a tensor $F_i^h(x)$ defined in the space V_n satisfies certain conditions, then such spaces are said to be endowed with an affinor structure [16], or equivalently, that an f-structure is defined in them [17].

Definition 2.1. The *degree* of an affinor structure defined in V_n is the smallest number k such that

$$F^h_{\alpha_1} F^{\alpha_1}_{\alpha_2} F^{\alpha_2}_{\alpha_3} \dots F^{\alpha_{k-2}}_{\alpha_{k-1}} F^{\alpha_{k-1}}_i := \overset{k}{F}{}^h_i = e\delta^h_i,$$

where $e \in \{-1, 0, 1\}$, and in the case of $e = 1$ it will be a *hyperbolic* f-structure, in the case of $e = 0$ it will be a *parabolic* f-structure, and in the case of $e = -1$ it will be an *elliptic* f-structure of the k-th degree [1], respectively.

3. Affinor-semisymmetric spaces

Consider a pseudo-Riemannian space V_n with a metric tensor $g_{ij}(x)$, in which a tensor $F_i^h(x)$ is defined [18].

For the sake of convenience, we introduce an operation of conjugation in V_n [19]:

$$A^{\cdots}_{\bar{i}\ldots} := A^{\cdots}_{\alpha\ldots} F^{\alpha}_{\cdot i} \qquad B^{\bar{i}\ldots}_{\ldots} := B^{\alpha\ldots}_{\ldots} F^{i}_{\alpha\cdot}, \tag{3.1}$$

where A and B are arbitrary tensors of any valence [11].

Definition 3.1. An arbitrary tensor $A^{j_1\ldots j_k}_{i_1\ldots i_l}$ is called *semisymmetric* [20] if the alternation of its first and second indices of the covariant derivative equals zero:

$$A^{j_1\ldots j_k}_{i_1\ldots i_l,[i_{l+1}i_{l+2}]} = 0,$$

where «,» is the symbol for the covariant derivative [21] in the space V_n.

Definition 3.2. The space V_n is called an *affinor-semisymmetric space* [25] if a semisymmetric affinor structure $F_i^h(x)$ is defined in it, i.e., the affinor $F_i^h(x)$ satisfies the conditions

$$F_{ij,[kl]} = 0. \tag{3.2}$$

Here, the index is lowered using the metric tensor

$$F_{ij} = g_{i\alpha} F^{\alpha}_{\cdot j}.$$

Consider a pseudo-Riemannian space V_n in which an elliptic affinor-semisymmetric structure of the second degree [12] is defined, i.e.,

$$F^{h}_{\cdot\alpha} F^{\alpha}_{\cdot i} = -\delta^h_i, \tag{3.3}$$

which additionally satisfies

$$F_{(ij)} = 0, \tag{3.4}$$

where the parentheses in the indices denote the operation of symmetrization, and δ^h_i are the Kronecker symbols [4].

Affinor structures that satisfy (3.3) are called *almost complex* [14]. By (3.1), formula (3.3) in terms of the conjugation operation takes the form

$$F^{h}_{\cdot\bar{i}} = -\delta^h_i.$$

We write the integrability conditions of the affinor [23], using the Ricci identity:

$$F_{ij,kl} - F_{ij,lk} = F_{\alpha j} R^{\alpha}_{\cdot ikl} + F_{i\alpha} R^{\alpha}_{\cdot jkl}.$$

By the antisymmetry of the affinor, we obtain

$$F_{\alpha j}R^{\alpha}_{\cdot ikl} + F_{i\alpha}R^{\alpha}_{\cdot jkl} = F_{\alpha j}R^{\alpha}_{\cdot ikl} - F_{\alpha i}R^{\alpha}_{\cdot jkl},$$

thus, the integrability conditions of the affinor [15] in terms of the conjugation operation are written in the form

$$F_{ij,kl} - F_{ij,lk} = R_{\bar{j}ikl} - R_{\bar{i}jkl},$$

or, taking into account the antisymmetry of the curvature tensor,

$$F_{ij,kl} - F_{ij,lk} = -(R_{\bar{i}jkl} + R_{i\bar{j}kl}). \tag{3.5}$$

Here we use (3.2)

$$R_{\bar{i}jkl} = -R_{i\bar{j}kl}. \tag{3.6}$$

From the symmetry property of the curvature tensor it follows that

$$R_{kl\bar{i}j} = -R_{kli\bar{j}}.$$

Contracting with the affinor over the index j, taking into account the almost complex property (3.3) of the space and performing an index substitution in terms of the conjugation operation, we obtain

$$R_{ij\bar{k}\bar{l}} = R_{ijkl}.$$

On the other hand, if it is known that in some space endowed with an antisymmetric affinor structure condition (3.6) holds, then such a space will be affinor-semisymmetric. Indeed, substituting (3.6) into (3.5) yields (3.2). Thus, the following has been proven:

Theorem 3.3. *A space V_n endowed with an antisymmetric affinor structure is affinor-semisymmetric if and only if condition* (3.6) *is satisfied.*

By virtue of (3.3), the following properties hold:

$$A^{\cdots}_{\bar{\bar{i}}\ldots} = -A^{\cdots}_{i\ldots},$$

$$B^{\bar{\bar{i}}\ldots}_{\ldots} = -B^{i\ldots}_{\ldots},$$

and by virtue of (3.4), the following holds:

$$A^{\cdots}_{\bar{\alpha}\ldots}B^{\alpha\ldots}_{\ldots} = A^{\cdots}_{\beta\ldots}F^{\beta}_{\cdot\alpha}B^{\alpha\ldots}_{\ldots} = -A^{\cdots}_{\beta\ldots}F^{\beta}_{\alpha\cdot}B^{\alpha\ldots}_{\ldots} =$$

$$= -A^{\cdots}_{\beta\ldots}B^{\bar{\beta}\ldots}_{\ldots} = -A^{\cdots}_{\alpha\ldots}B^{\bar{\alpha}\ldots}_{\ldots},$$

Therefore, taking into account the last two properties, we have

$$A^{\cdots}_{\bar{\alpha}\ldots}B^{\bar{\alpha}\ldots}_{\ldots} = A^{\cdots}_{\alpha\ldots}B^{\alpha\ldots}_{\ldots}.$$

The metric tensor satisfies the relations [26]

$$g_{\bar{i}\bar{j}} = g_{\alpha\beta}F^{\alpha}_{\cdot i}F^{\beta}_{\cdot j} = F_{\beta i}F^{\beta}_{\cdot j} \overset{(3.4)}{=} -F_{i\beta}F^{\beta}_{\cdot j} \overset{(3.3)}{=} g_{ij} \tag{3.7}$$

$$g_{\bar{i}j} = g_{\alpha j}F^{\alpha}_{\cdot i} = F_{ji} \overset{(3.4)}{=} -F_{ij} = -g_{i\alpha}F^{\alpha}_{\cdot j} = -g_{i\bar{j}}.$$

The Kronecker symbols satisfy the relation

$$\delta^{h}_{\bar{i}} = \delta^{h}_{\alpha} F^{\alpha}_{\cdot i} = F^{h}_{\cdot i} = -F^{h}_{i\cdot} = -\delta^{\alpha}_{i} F^{h}_{\alpha} = -\delta^{\bar{h}}_{i},$$

therefore

$$\delta^{\bar{h}}_{\bar{i}} = -\delta^{h}_{\bar{\bar{i}}} \overset{(3.4)}{=} \delta^{h}_{i}.$$

The Riemann tensor, in addition to the known identities [27], satisfies the following properties:

$$\begin{gathered} R_{\bar{h}\bar{i}jk} \overset{(3.6)}{=} -R_{\bar{\bar{h}}ijk} \overset{(3.3)}{=} R_{hijk}, \\ R_{\bar{i}jkl} = R_{kl\bar{i}j} = -R_{i\bar{j}kl} = -R_{kli\bar{j}}. \end{gathered}$$

Contracting (3.6) over the indices h, k gives

$$\begin{gathered} R_{\alpha ij\beta} F^{\alpha}_{\cdot\gamma} g^{\beta\gamma} = -R_{\bar{i}j} \\ R_{\alpha ij\beta} F^{\beta\alpha} = R_{\bar{i}j} \\ R^{\bar{\beta}}_{\cdot ij\beta} = R_{\bar{i}j} \end{gathered}$$

Further contracting the equality $R_{h\bar{i}\bar{j}k} = R_{\bar{h}ij\bar{k}}$ over the indices h, k, we get

$$R_{\bar{i}\bar{j}} = R_{\alpha ij\beta} F^{\alpha}_{\cdot\gamma} F^{\beta}_{\cdot\delta} g^{\gamma\delta} = -R_{\alpha ij\beta} F^{\alpha}_{\cdot\gamma} F^{\gamma\beta} = R_{ij},$$

therefore

$$R_{\bar{i}j} = -R_{i\bar{j}}.$$

Next, covariantly differentiate [28] equality (3.3) with respect to x^k:

$$\begin{gathered} F^{h}_{\cdot\alpha,k} F^{\alpha}_{\cdot i} + F^{h}_{\cdot\alpha} F^{\alpha}_{\cdot i,k} = 0, \\ F^{h}_{\cdot\alpha,k} F^{\alpha}_{\cdot i} - F^{h}_{\alpha\cdot} F^{\alpha}_{\cdot i,k} = 0, \\ F^{h}_{\cdot\bar{i},k} = F^{\bar{h}}_{\cdot i,k}. \end{gathered}$$

Similarly,

$$F_{\bar{h}i,k} = F_{h\bar{i},k}.$$

Covariantly differentiating this expression with respect to x^j and alternating over the indices k, j, we obtain

$$F_{\bar{h}i,[kj]} = F_{h\bar{i},[kj]}.$$

Thus, special properties of the affinor structure [29] and the intrinsic geometry [30] of affinor-semisymmetric spaces have been obtained.

4. Pseudo-Riemannian spaces of constant almost holomorphic curvature

Definition 4.1. A pseudo-Riemannian space V_n $(n > 2)$ with a metric tensor g_{ij} is called a *space of constant almost holomorphic curvature of type I* if its curvature tensor R_{hijk} satisfies the conditions

$$\begin{aligned} R_{hijk} = &-2\overset{1}{\tau}(g_{hj}g_{ki} - g_{hk}g_{ji} + g_{h\bar{j}}g_{\bar{k}i} - g_{h\bar{k}}g_{\bar{j}i} + 2g_{h\bar{i}}g_{\bar{j}k}) \\ &- \overset{2}{\tau}(g_{hj}u_k u_i - g_{hk}u_j u_i + g_{h\bar{j}}u_{\bar{k}}u_i - g_{h\bar{k}}u_{\bar{j}}u_i + 2g_{h\bar{i}}u_{\bar{j}}u_k \\ &+ g_{ki}u_h u_j - g_{ji}u_h u_k + g_{\bar{k}i}u_h u_{\bar{j}} - g_{\bar{j}i}u_h u_{\bar{k}} + 2g_{\bar{j}k}u_h u_{\bar{i}}), \end{aligned} \tag{4.1}$$

and *of type II* if its curvature tensor R_{hijk} satisfies the conditions

$$\begin{aligned} R_{hijk} = &-2\overset{3}{\tau}(g_{hj}g_{ki} - g_{hk}g_{ij} + g_{h\bar{j}}g_{\bar{k}i} - g_{h\bar{k}}g_{i\bar{j}} + 2g_{h\bar{i}}g_{\bar{j}k}) \\ &- \overset{4}{\tau}(g_{hj}(u_k u_i + u_{\bar{k}}u_{\bar{i}}) - g_{hk}(u_i u_j + u_{\bar{i}}u_{\bar{j}}) \\ &+ g_{h\bar{j}}(u_{\bar{k}}u_i - u_k u_{\bar{i}}) - g_{h\bar{k}}(u_i u_{\bar{j}} - u_{\bar{i}}u_j) \\ &+ 2g_{h\bar{i}}(u_{\bar{j}}u_k - u_j u_{\bar{k}}) + g_{ki}(u_h u_j + u_{\bar{h}}u_{\bar{j}}) \\ &- g_{ji}(u_h u_k + u_{\bar{h}}u_{\bar{k}}) + g_{\bar{k}i}(u_h u_{\bar{j}} - u_{\bar{h}}u_j) \\ &- g_{\bar{j}i}(u_h u_{\bar{k}} - u_{\bar{h}}u_k) + 2g_{\bar{j}k}(u_h u_{\bar{i}} - u_{\bar{h}}u_i)), \end{aligned} \tag{4.2}$$

where $\overset{1}{\tau}, \overset{2}{\tau}, \overset{3}{\tau}, \overset{4}{\tau}$ are some non-zero invariants, and u_i is some non-zero vector.

This definition uses the conjugation operation, meaning an affinor structure is defined in the space. Let the affinor structure of this space be elliptic antisymmetric of the second degree. Also, for spaces of type II, the expression

$$u_{\bar{i}}u_{\bar{j}} = \kappa u_i u_j$$

obviously does not hold for any invariant κ, otherwise it would be a space of type I.

We now turn to spaces of type I. Contracting equality (4.1) over the indices h, k, we obtain the Ricci tensor of this space:

$$R_{ij} = 2\overset{1}{\tau}(n-2)g_{ji} + \overset{2}{\tau}((n-1)u_i u_j - 3u_{\bar{i}}u_{\bar{j}} + g_{ji}u^{\alpha}u_{\alpha}). \tag{4.3}$$

Conjugating this expression with respect to the indices i, j and taking into account (3.7) and the relation $R_{\bar{i}\bar{j}} = R_{ij}$, we have

$$R_{ij} = 2\overset{1}{\tau}(n-2)g_{ji} + \overset{2}{\tau}((n-1)u_{\bar{i}}u_{\bar{j}} - 3u_i u_j + g_{ji}u^{\alpha}u_{\alpha})$$

and subtract it from (4.3), since $\overset{2}{\tau} \neq 0$

$$u_{\bar{i}}u_{\bar{j}} = u_i u_j. \tag{4.4}$$

Contracting the resulting equality with u^j yields

$$u_i u_\alpha u^\alpha = 0. \tag{4.5}$$

Since $u_i \neq 0$, the last equality indicates the fulfillment of

Theorem 4.2. *In spaces of constant almost holomorphic curvature of type I with an elliptic antisymmetric affinor structure of the second degree, the vector u_i is isotropic.*

Based on this theorem, taking into account (4.4), (4.5), the identity (4.3) takes the form

$$R_{ij} = 2\overset{1}{\tau}(n-2)g_{ji} + \overset{2}{\tau}(n-4)u_i u_j. \tag{4.6}$$

Hence, the integrability conditions for the Ricci tensor, taking into account the Ricci identity, have the form

$$R_{ij,[kl]} = R_{\alpha j} R^\alpha_{ikl} + R_{i\alpha} R^\alpha_{jkl},$$

or, expanding the Riemann tensor by definition,

$$R_{ij,[kl]} = 0.$$

It is proven:

Theorem 4.3. *Spaces of constant almost holomorphic curvature of type I with an elliptic antisymmetric affinor structure of the second degree are Ricci-semisymmetric.*

From (4.6) for $n \neq 4$ and $\overset{2}{\tau} \neq 0$ it follows that

$$u_i u_j = \frac{1}{\overset{2}{\tau}(n-4)}(R_{ij} - 2\overset{1}{\tau}(n-2)g_{ji}),$$

therefore, the curvature tensor can be rewritten in the form

$$\begin{aligned} R_{hijk} = -\frac{1}{n-4}(&g_{hj}\hat{R}_{ki} - g_{hk}\hat{R}_{ij} + g_{h\bar{j}}\hat{R}_{\bar{k}i} - g_{h\bar{k}}\hat{R}_{i\bar{j}} + 2g_{h\bar{i}}\hat{R}_{\bar{j}k} \\ &+ R_{hj}g_{ki} - R_{hk}g_{ij} + R_{h\bar{j}}g_{\bar{k}i} - R_{h\bar{k}}g_{i\bar{j}} + 2R_{h\bar{i}}g_{\bar{j}k}), \end{aligned}$$

where

$$\hat{R}_{ij} := R_{ij} - 2n\overset{1}{\tau}g_{ij}.$$

We write the integrability conditions for the curvature tensor, taking into account the Ricci identity and Ricci-semisymmetry:

$$R_{hijk,[lm]} = 0.$$

It is proven:

Corollary 4.4. *Spaces of constant almost holomorphic curvature of type I with a semisymmetric elliptic antisymmetric affinor structure of the second degree are semisymmetric.*

Multiplying both sides of equality (4.6) by g^{ij} and contracting over the indices i, j, we obtain

$$R = 2\overset{1}{\tau}n(n-2) + \overset{2}{\tau}(n-4)u_\alpha u^\alpha$$

and express $\overset{1}{\tau}$, since $n > 2$

$$\overset{1}{\tau} = \frac{R - \overset{2}{\tau}(n-4)u_\alpha u^\alpha}{2n(n-2)}.$$

Rewriting (4.6) gives

$$R_{ij} = \frac{R}{n}g_{ij} - \frac{\overset{2}{\tau}(n-4)u_\alpha u^\alpha}{n}g_{ij} + \overset{2}{\tau}(n-4)u_i u_j.$$

From the last equality, it is evident that for Einstein spaces [31], i.e., spaces where the conditions $R_{ij} = \frac{R}{n}g_{ji}$ hold, the invariant $\overset{2}{\tau}$ is necessarily zero, and the space of constant almost holomorphic curvature will be a space of constant curvature. Therefore, in what follows, we will consider spaces that are not Einstein spaces.

Definition 4.5. A pseudo-Riemannian space whose Ricci tensor can be represented in the form

$$R_{ij} = \frac{R}{n}g_{ij} + U_i U_j,$$

where U_i is an isotropic gradient vector, is called an *almost Einstein space*.

Corollary 4.6. *Spaces of constant almost holomorphic curvature of type I with an elliptic antisymmetric affinor structure of the second degree are almost Einstein spaces, where*

$$U_i = \frac{\overset{2}{\tau}(n-4)}{2}u_i.$$

Almost Einstein spaces are well studied, therefore further research will be conducted for spaces of type II.

We now turn to spaces of type II. Contracting the Riemann tensor over the indices h, k, we obtain the Ricci tensor of this space:

$$R_{ij} = 2\overset{3}{\tau}(n-2)g_{ij} + \overset{4}{\tau}(n-4)(u_i u_j + u_{\bar{i}} u_{\bar{j}}). \tag{4.7}$$

Hence, the integrability conditions for the Ricci tensor, taking into account the Ricci identity, have the form

$$R_{ij,[kl]} = R_{\alpha j}R^\alpha_{ikl} + R_{i\alpha}R^\alpha_{jkl},$$

or, expanding the Riemann tensor by definition,

$$R_{ij,[kl]} = 0.$$

It is proven:

Theorem 4.7. *Spaces of constant almost holomorphic curvature of type II with a semisymmetric elliptic antisymmetric affinor structure of the second degree are Ricci-semisymmetric.*

From the Ricci tensor, it is evident that for $n \neq 4$ and $\overset{4}{\tau} \neq 0$,

$$u_i u_j + u_{\bar{i}} u_{\bar{j}} = \frac{1}{\overset{4}{\tau}(n-4)}(R_{ij} - 2\overset{3}{\tau}(n-2)g_{ij}),$$

therefore, substituting this into (4.2) yields another representation for the Riemann tensor:

$$R_{hijk} = -\frac{1}{n-4}(g_{hj}\hat{R}_{ki} - g_{hk}\hat{R}_{ij} + g_{h\bar{j}}\hat{R}_{\bar{k}i} - g_{h\bar{k}}\hat{R}_{i\bar{j}} + 2g_{h\bar{i}}\hat{R}_{\bar{j}k} \\ + R_{hj}g_{ki} - R_{hk}g_{ij} + R_{h\bar{j}}g_{\bar{k}i} - R_{h\bar{k}}g_{i\bar{j}} + 2R_{h\bar{i}}g_{\bar{j}k}),$$

where

$$\hat{R}_{ij} := R_{ij} - 2n\overset{3}{\tau}g_{ij}.$$

We write the integrability conditions for the curvature tensor, taking into account the Ricci identity and Ricci-semisymmetry:

$$R_{hijk,[lm]} = 0.$$

It is proven:

Corollary 4.8. *Spaces of constant almost holomorphic curvature of type II with a semisymmetric elliptic antisymmetric affinor structure of the second degree are semisymmetric.*

Next, contracting the Ricci tensor over the indices i, j yields the scalar curvature of the space:

$$R = 2\overset{3}{\tau}n(n-2) + 2\overset{4}{\tau}(n-4)u_\alpha u^\alpha. \tag{4.8}$$

From this equality, we express $\overset{3}{\tau}$, since $n > 2$:

$$\overset{3}{\tau} = \frac{R - 2\overset{4}{\tau}(n-4)u_\alpha u^\alpha}{2n(n-2)}$$

and substitute it into the Ricci tensor:

$$R_{ij} = \frac{R}{n}g_{ij} - \frac{2\overset{4}{\tau}(n-4)u_\alpha u^\alpha}{n}g_{ij} + \overset{4}{\tau}(n-4)(u_i u_j + u_{\bar{i}} u_{\bar{j}}).$$

From the last equality, it is evident that for Einstein spaces, i.e., spaces where the conditions $R_{ij} = \frac{R}{n}g_{ij}$ hold, the invariant $\overset{4}{\tau}$ is necessarily zero, and the space of constant almost holomorphic curvature will be a space of constant curvature. Therefore, in what follows, we will consider spaces that are not Einstein spaces.

Definition 4.9. A pseudo-Riemannian space whose Riemann tensor can be represented in the form

$$R^h_{ijk} = -(\delta^h_{[j} B_{k]i} + \delta^h_{[\bar{j}} B_{\bar{k}]i} + 2\delta^h_{\bar{i}} B_{\bar{j}k} + \\ + B^h_{[j} g_{k]i} + B^h_{[\bar{j}} g_{\bar{k}]i} + 2B^h_{\bar{i}} g_{\bar{j}k}),$$

where B_{ij} is an arbitrary twice-covariant tensor, is called a *Bochner-flat space*.

From this definition, it follows that

Theorem 4.10. *Spaces of constant almost holomorphic curvature are Bochner-flat, where for type I spaces*

$$B_{ij} = \overset{1}{\tau} g_{ij} + \overset{2}{\tau} u_i u_j,$$

and for type II spaces

$$B_{ij} = \overset{3}{\tau} g_{ij} + \overset{4}{\tau}(u_i u_j + u_{\bar{i}} u_{\bar{j}}).$$

5. Reducible spaces with constant almost holomorphic curvature of type II

Definition 5.1. Let V_n be a pseudo-Riemannian space with a metric tensor g_{ij}, $n > 2$. A Riemannian space V_n is called (locally) reducible [4] if, in some neighborhood D of each of its points M, a coordinate system $y^1, y^2, \ldots, y^n$ can be chosen such that the fundamental metric form has the form:

$$I = g_{ab}(x^c)dx^a dx^b + g_{\alpha\beta}(x^\gamma)dx^\alpha dx^\beta, \\ a, b, c = \overline{1,\, k}, \quad \alpha,\, \beta,\, \gamma = \overline{k+1,\, n}.$$

Here, g_{ab} are assumed to depend only on $x^1, x^2, \ldots, x^k$, and $g_{\alpha\beta}$ – only on $x^{k+1}, x^{k+2}, \ldots, x^n$. Thus, according to the definition, a reducible Riemannian space V_n represents the product of a Riemannian space V_m (referred to the coordinates $x^1, x^2, \ldots, x^k$) with the fundamental metric form $g_{ab}(x^c)dx^a dx^b$ and a Riemannian space V_{n-m} (referred to the coordinates $x^{k+1}, x^{k+2}, \ldots, x^n$) with the metric form $g_{\alpha\beta}(x^\gamma)dx^\alpha dx^\beta$. Therefore, the matrix of the metric tensor has the form

$$g_{ij} = \begin{pmatrix} g_{11} & \dots & g_{1k} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ g_{k1} & \dots & g_{kk} & 0 & \dots & 0 \\ 0 & \dots & 0 & g_{k+1k+1} & \dots & g_{k+1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & g_{nk+1} & \dots & g_{nn} \end{pmatrix}.$$

Definition 5.2. Twice-covariant tensors that satisfy the relation

$$a_{i\alpha} A^{\alpha k_1 \dots k_p}_{jl_1 \dots l_q} = a_{j\alpha} A^{\alpha k_1 \dots k_p}_{il_1 \dots l_q},$$

where $A^{kk_1 \dots k_p}_{il_1 \dots l_q}$ is some tensor of arbitrary rank $\binom{p+1}{q+1}$, are called *commuting with the tensor* $A^{kk_1 \dots k_p}_{il_1 \dots l_q}$.

Lemma 5.3. *For a pseudo-Riemannian space to be reducible, it is necessary and sufficient that there exists a twice-covariant symmetric idempotent covariantly constant tensor a_{ij} which is not proportional to the metric tensor* [5]*:*

$$a_{ij,k} = 0, \qquad a_{ij} = a_{ji},$$
$$a_{i\alpha} a^{\alpha}_{j} = a_{ij}, \qquad a_{ij} \neq c g_{ij}.$$

The integrability conditions for a_{ij} have the form:

$$a_{ij,[kl]} = a_{\alpha j} R^{\alpha}_{ikl} + a_{i\alpha} R^{\alpha}_{jkl} = 0.$$

Cyclically permuting the indices i, l, we obtain

$$a_{\alpha i} R^{\alpha}_{jkl} + a_{\alpha k} R^{\alpha}_{jli} + a_{\alpha l} R^{\alpha}_{jik} = 0.$$

From the latter, contracting with g^{ij}, we get

$$a_{\alpha l} R^{\alpha}_{k} - a_{\alpha k} R^{\alpha}_{l} = 0.$$

Substituting condition (4.7) into this equality, we verify that for $\overset{4}{\tau} \neq 0$,

$$a^{\alpha}_{l}(u_\alpha u_k + u_{\bar\alpha} u_{\bar k}) - a^{\alpha}_{k}(u_\alpha u_l + u_{\bar\alpha} u_{\bar l}) = 0. \tag{5.1}$$

Contracting this expression with a^{l}_{j}, taking into account the idempotency of the tensor a_{ij}, and performing an index substitution from j to l, we obtain:

$$a^{\alpha}_{l}(u_\alpha u_k + u_{\bar\alpha} u_{\bar k}) - a^{\alpha}_{k}(u_\alpha u_\beta a^{\beta}_{l} + u_{\bar\alpha} u_{\bar\beta} a^{\beta}_{l}) = 0. \tag{5.2}$$

Subtracting the obtained identity from (5.1), we get

$$a^{\alpha}_{k}(u_\alpha u_\beta a^{\beta}_{l} + u_{\bar\alpha} u_{\bar\beta} a^{\beta}_{l}) - a^{\alpha}_{k}(u_\alpha u_l + u_{\bar\alpha} u_{\bar l}) = 0$$

and contract with u^l

$$a^\alpha_k u_\alpha (u_\beta a^\beta_\gamma u^\gamma - u_\beta u^\beta) = -a^\alpha_k u_{\bar\alpha} u_{\bar\beta} a^\beta_\gamma u^\gamma,$$

then, if $u_\beta a^\beta_\gamma u^\gamma \neq u_\beta u^\beta$, we obtain

$$a^\alpha_k u_\alpha = \hat u a^\alpha_k u_{\bar\alpha}, \tag{5.3}$$

where

$$\hat u := -\frac{u_{\bar\beta} a^\beta_\gamma u^\gamma}{u_\beta a^\beta_\gamma u^\gamma - u_\beta u^\beta}.$$

Using $a^\alpha_k u_\alpha \neq 0$, $1 + \hat u^2 \neq 1$, $\hat u \neq 0$ and (5.3) in (5.2), we obtain

$$a^\alpha_l u_\alpha = \frac{1}{1+\hat u^2}(u_l + \hat u u_{\bar l}) \tag{5.4}$$

and

$$a^\alpha_l u_{\bar\alpha} = \frac{1}{1+\hat u^2}\left(\frac{1}{\hat u} u_l + u_{\bar l}\right). \tag{5.5}$$

Taking into account equalities (5.4) and (5.5) in (5.1), we obtain

$$(u_{\bar l} u_k - u_{\bar k} u_l)\left(\hat u - \frac{1}{\hat u}\right) = 0.$$

Here, if $|\hat u| \neq 1$, conjugate with respect to index k

$$u_{\bar k} u_{\bar l} = -u_k u_l,$$

which contradicts the definition of spaces of constant almost holomorphic curvature of type II. Thus, $|\hat u| = 1$. We examine the cases $|\hat u| = 1$.

Let $\hat u = -1$. In this case, the following will hold:

$$a^\alpha_l u_{\bar\alpha} = -a^\alpha_l u_\alpha = \frac{1}{2}(u_{\bar l} - u_l) \tag{5.6}$$

Substitute the Riemann tensor of the space of constant almost holomorphic curvature of type II into the integrability conditions of the tensor a_{ij}

$$\begin{aligned}
&2\overset{3}{\tau}(a_{jk} g_{li} - a_{jl} g_{ik} + a_{j\bar k} g_{\bar l i} - a_{j\bar l} g_{i\bar k} + 2a_{j\bar i} g_{\bar k l})\\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar l} u_{\bar i}) - a_{jl}(u_i u_k + u_{\bar i} u_{\bar k}) + a_{j\bar k}(u_{\bar l} u_i - u_l u_{\bar i})\\
&- a_{j\bar l}(u_i u_{\bar k} - u_{\bar i} u_k) + 2a_{j\bar i}(u_{\bar k} u_l - u_k u_{\bar l})\\
&+ g_{li}(a_{\alpha j} u^\alpha u_k + a_{\alpha j} u^{\bar\alpha} u_{\bar k}) - g_{ki}(a_{\alpha j} u^\alpha u_l + a_{\alpha j} u^{\bar\alpha} u_{\bar l})\\
&+ g_{\bar l i}(a_{\alpha j} u^\alpha u_{\bar k} - a_{\alpha j} u^{\bar\alpha} u_k) - g_{\bar k i}(a_{\alpha j} u^\alpha u_{\bar l} - a_{\alpha j} u^{\bar\alpha} u_l)\\
&+ 2g_{\bar k l}(a_{\alpha j} u^\alpha u_{\bar i} - a_{\alpha j} u^{\bar\alpha} u_i)) + (i \leftrightarrow j) = 0,
\end{aligned}$$

or, taking into account (5.6)

$$\begin{aligned}
&2\overset{3}{\tau}(a_{jk}g_{li} - a_{jl}g_{ik} + a_{j\bar{k}}g_{\bar{l}i} - a_{j\bar{l}}g_{i\bar{k}} + 2a_{j\bar{i}}g_{\bar{k}l})\\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar{l}}u_{\bar{i}}) - a_{jl}(u_i u_k + u_{\bar{i}}u_{\bar{k}}) + a_{j\bar{k}}(u_{\bar{l}}u_i - u_l u_{\bar{i}})\\
&- a_{j\bar{l}}(u_i u_{\bar{k}} - u_{\bar{i}}u_k) + 2a_{j\bar{i}}(u_{\bar{k}}u_l - u_k u_{\bar{l}})\\
&+ a_{\alpha j}u^{\alpha}(g_{li}(u_k - u_{\bar{k}}) - g_{ki}(u_l - u_{\bar{l}})\\
&+ g_{\bar{l}i}(u_{\bar{k}} + u_k) - g_{\bar{k}i}(u_{\bar{l}} + u_l)\\
&+ 2g_{\bar{k}l}(u_{\bar{i}} + u_i))) + (i \leftrightarrow j) = 0.
\end{aligned}$$

We rewrite this identity as

$$\begin{aligned}
&g_{li}A_{jk} - g_{ik}A_{jl} + g_{\bar{l}i}A_{j\bar{k}} - g_{i\bar{k}}A_{j\bar{l}} + 2g_{\bar{k}l}A_{j\bar{i}}\\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar{l}}u_{\bar{i}}) - a_{jl}(u_i u_k + u_{\bar{i}}u_{\bar{k}}) + a_{j\bar{k}}(u_{\bar{l}}u_i - u_l u_{\bar{i}})\\
&- a_{j\bar{l}}(u_i u_{\bar{k}} - u_{\bar{i}}u_k) + 2a_{j\bar{i}}(u_{\bar{k}}u_l - u_k u_{\bar{l}})) + (i \leftrightarrow j) = 0,
\end{aligned} \tag{5.7}$$

where

$$A_{ij} := 2\overset{3}{\tau}a_{ij} + \frac{\overset{4}{\tau}}{2}(u_i - u_{\bar{i}})(u_j - u_{\bar{j}}). \tag{5.8}$$

Note that A_{ij} is necessarily symmetric. Multiplying equality (5.7) by g^{jk} and summing over the indices j, k, we obtain

$$\begin{aligned}
&g_{li}A - A_{\bar{l}\bar{i}} + (1-n)A_{il} + \overset{4}{\tau}(a(u_l u_i + u_{\bar{l}}u_{\bar{i}})\\
&- 2u_{\alpha}u^{\alpha}a_{il} - (u_i u_{\bar{l}} + u_{\bar{i}}u_l)) = 0,
\end{aligned} \tag{5.9}$$

Conjugating the resulting expression with respect to the indices i, l gives

$$\begin{aligned}
&g_{li}A - A_{li} + (1-n)A_{\bar{i}\bar{l}} + \overset{4}{\tau}(a(u_l u_i + u_{\bar{l}}u_{\bar{i}})\\
&- 2u_{\alpha}u^{\alpha}a_{\bar{i}\bar{l}} + (u_i u_{\bar{l}} + u_{\bar{i}}u_l)) = 0,
\end{aligned}$$

and subtract it from (5.9)

$$(n-2)(A_{\bar{i}\bar{l}} - A_{il}) + 2\overset{4}{\tau}(u_{\alpha}u^{\alpha}(a_{\bar{i}\bar{l}} - a_{il}) - (u_i u_{\bar{l}} + u_{\bar{i}}u_l)) = 0. \tag{5.10}$$

Contracting this identity with u^l and using $n > 2$, we obtain

$$A_{\bar{i}\bar{\alpha}}u^{\alpha} = A_{i\alpha}u^{\alpha}.$$

Expanding A_{ij} by definition, we get

$$\overset{3}{\tau} = -\frac{u_{\alpha}u^{\alpha}}{2}\overset{4}{\tau}. \tag{5.11}$$

Note that the invariant $u_{\alpha}u^{\alpha} \neq 0$, otherwise $\overset{3}{\tau} = 0$ would hold, which is impossible. Thus, it is proven:

Lemma 5.4. *In reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree for $\hat{u} = -1$, the vector u_i is non-isotropic.*

Next, expanding A_{ij} by definition in (5.10) and using (5.11) and $\overset{3}{\tau} \neq 0$, we obtain:

$$-u_\alpha u^\alpha (a_{\bar{i}\bar{l}} - a_{il}) + u_i u_{\bar{l}} + u_{\bar{i}} u_l = 0, \tag{5.12}$$

and, therefore,

$$A_{\bar{i}\bar{l}} = A_{il}.$$

Taking this into account in (5.9) for $n > 2$, $u_\alpha u^\alpha \neq 0$, we obtain:

$$a_{il} = g_{il}\frac{a-1}{n-2} - \frac{1}{(n-2)u_\alpha u^\alpha}((a - \frac{n}{2})(u_l u_i + u_{\bar{l}} u_{\bar{i}}) \\ - (1 - \frac{n}{2})(u_i u_{\bar{l}} + u_{\bar{i}} u_l)),$$

Contracting this expression with a^l_j, taking into account the idempotency of the tensor a_{ij}, and performing an index substitution $j \to l$, we obtain:

$$a_{il} = \frac{-1}{2u_\alpha u^\alpha}(u_i - u_{\bar{i}})(u_l - u_{\bar{l}}). \tag{5.13}$$

Multiplying and dividing this expression by $\overset{4}{\tau}$, covariantly differentiating with respect to x^j and taking into account the covariant constancy of a_{ij}, we obtain that the expression is covariantly constant:

$$(\overset{4}{\tau}(u_i - u_{\bar{i}})(u_l - u_{\bar{l}}))_{,j} = 0$$

and, therefore,

$$A_{ij,k} = 0.$$

Substituting (5.13) into (5.8) and using (5.11), we obtain:

$$A_{ij} = -(\frac{1}{\overset{4}{\tau} u_\alpha u^\alpha} + \overset{4}{\tau} u_\alpha u^\alpha) a_{ij}. \tag{5.14}$$

Conjugating this expression over all indices and subtracting it from (5.14), we get:

$$(\frac{1}{\overset{4}{\tau} u_\alpha u^\alpha} + \overset{4}{\tau} u_\alpha u^\alpha)(a_{ij} - a_{\bar{i}\bar{j}}) = 0.$$

Here, one of the factors is zero, and if it is the second factor, i.e., $a_{ij} = a_{\bar{i}\bar{j}}$, then substituting this into (5.12) we find that $u_{\bar{i}} u_{\bar{j}} = u_i u_j$, which is impossible. On the other hand, the first factor has no roots over the field of real numbers. It is proven:

Theorem 5.5. *In reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree, the invariant* $\hat{u} \neq -1$.

Based on this theorem, we will further assume $\hat{u} = 1$. Then the following will hold:

$$a_l^\alpha u_\alpha = a_l^\alpha u_{\bar{\alpha}} = \frac{1}{2}(u_l + u_{\bar{l}}) \tag{5.15}$$

Substituting the Riemann tensor of the space of constant almost holomorphic curvature of type II into the integrability conditions of the tensor a_{ij}, we obtain

$$\begin{aligned}
&2\overset{3}{\tau}(a_{jk}g_{li} - a_{jl}g_{ik} + a_{j\bar{k}}g_{\bar{l}i} - a_{j\bar{l}}g_{i\bar{k}} + 2a_{j\bar{i}}g_{\bar{k}l}) \\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar{l}}u_{\bar{i}}) - a_{jl}(u_i u_k + u_{\bar{i}}u_{\bar{k}}) + a_{j\bar{k}}(u_{\bar{l}}u_i - u_l u_{\bar{i}}) \\
&- a_{j\bar{l}}(u_i u_{\bar{k}} - u_{\bar{i}}u_k) + 2a_{j\bar{i}}(u_{\bar{k}}u_l - u_k u_{\bar{l}}) \\
&+ g_{li}(a_{\alpha j}u^\alpha u_k + a_{\alpha j}u^{\bar{\alpha}}u_{\bar{k}}) - g_{ki}(a_{\alpha j}u^\alpha u_l + a_{\alpha j}u^{\bar{\alpha}}u_{\bar{l}}) \\
&+ g_{\bar{l}i}(a_{\alpha j}u^\alpha u_{\bar{k}} - a_{\alpha j}u^{\bar{\alpha}}u_k) - g_{\bar{k}i}(a_{\alpha j}u^\alpha u_{\bar{l}} - a_{\alpha j}u^{\bar{\alpha}}u_l) \\
&+ 2g_{\bar{k}l}(a_{\alpha j}u^\alpha u_{\bar{i}} - a_{\alpha j}u^{\bar{\alpha}}u_i)) + (i \leftrightarrow j) = 0,
\end{aligned}$$

or, taking into account (5.15)

$$\begin{aligned}
&2\overset{3}{\tau}(a_{jk}g_{li} - a_{jl}g_{ik} + a_{j\bar{k}}g_{\bar{l}i} - a_{j\bar{l}}g_{i\bar{k}} + 2a_{j\bar{i}}g_{\bar{k}l}) \\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar{l}}u_{\bar{i}}) - a_{jl}(u_i u_k + u_{\bar{i}}u_{\bar{k}}) + a_{j\bar{k}}(u_{\bar{l}}u_i - u_l u_{\bar{i}}) \\
&- a_{j\bar{l}}(u_i u_{\bar{k}} - u_{\bar{i}}u_k) + 2a_{j\bar{i}}(u_{\bar{k}}u_l - u_k u_{\bar{l}}) \\
&+ \frac{1}{2}(u_j + u_{\bar{j}})(g_{li}(u_k + u_{\bar{k}}) - g_{ki}(u_l + u_{\bar{l}}) \\
&+ g_{\bar{l}i}(u_{\bar{k}} - u_k) - g_{\bar{k}i}(u_{\bar{l}} - u_l) \\
&+ 2g_{\bar{k}l}(u_{\bar{i}} - u_i))) + (i \leftrightarrow j) = 0.
\end{aligned}$$

We rewrite this identity as

$$\begin{aligned}
&g_{li}A_{jk} - g_{ik}A_{jl} + g_{\bar{l}i}A_{j\bar{k}} - g_{i\bar{k}}A_{j\bar{l}} + 2g_{\bar{k}l}A_{j\bar{i}} \\
&+ \overset{4}{\tau}(a_{jk}(u_l u_i + u_{\bar{l}}u_{\bar{i}}) - a_{jl}(u_i u_k + u_{\bar{i}}u_{\bar{k}}) + a_{j\bar{k}}(u_{\bar{l}}u_i - u_l u_{\bar{i}}) \\
&- a_{j\bar{l}}(u_i u_{\bar{k}} - u_{\bar{i}}u_k) + 2a_{j\bar{i}}(u_{\bar{k}}u_l - u_k u_{\bar{l}})) + (i \leftrightarrow j) = 0,
\end{aligned} \tag{5.16}$$

where

$$A_{ij} \coloneqq 2\overset{3}{\tau}a_{ij} + \frac{\overset{4}{\tau}}{2}(u_i + u_{\bar{i}})(u_j + u_{\bar{j}}). \tag{5.17}$$

Note that A_{ij} is necessarily symmetric. Multiplying equality (5.16) by g^{jk} and summing over the indices j, k, we obtain

$$\begin{aligned} & g_{li}A + (1-n)A_{il} - A_{\bar{l}\bar{i}} + \\ & + \overset{4}{\tau}(a(u_l u_i + u_{\bar{l}} u_{\bar{i}}) + u_{\bar{i}} u_l + u_i u_{\bar{l}} - 2a_{il} u_\alpha u^\alpha) = 0. \end{aligned} \tag{5.18}$$

Conjugating the resulting expression with respect to the indices i, l gives

$$\begin{aligned} & g_{li}A + (1-n)A_{\bar{i}\bar{l}} - A_{li} + \\ & + \overset{4}{\tau}(a(u_l u_i + u_{\bar{l}} u_{\bar{i}}) - u_{\bar{i}} u_l - u_i u_{\bar{l}} - 2a_{\bar{i}\bar{l}} u_\alpha u^\alpha) = 0 \end{aligned}$$

and subtract it from (5.18)

$$(n-2)(A_{\bar{i}\bar{l}} - A_{il}) + 2\overset{4}{\tau}(u_{\bar{i}} u_l + u_i u_{\bar{l}} + u_\alpha u^\alpha (a_{\bar{i}\bar{l}} - a_{il})) = 0. \tag{5.19}$$

Contracting this identity with u^l yields

$$A_{\bar{i}\bar{\alpha}} u^\alpha = A_{i\alpha} u^\alpha.$$

Expanding A_{ij} by definition, we obtain

$$\overset{3}{\tau} = -\frac{u_\alpha u^\alpha}{2} \overset{4}{\tau}. \tag{5.20}$$

Note that the invariant $u_\alpha u^\alpha \neq 0$, otherwise $\overset{3}{\tau} = 0$ would hold, which is impossible. Thus, it is proven:

Lemma 5.6. *In reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree, the vector u_i is non-isotropic.*

Next, expanding A_{ij} by definition in (5.19) and using (5.20), $n > 2$, we obtain:

$$u_\alpha u^\alpha (a_{\bar{i}\bar{l}} - a_{il}) + u_i u_{\bar{l}} + u_{\bar{i}} u_l = 0, \tag{5.21}$$

and, therefore,

$$A_{\bar{i}\bar{l}} = A_{il}.$$

Taking this into account in (5.18) and using the non-isotropy of the vector u_i and $n > 2$, we prove the following:

Theorem 5.7. *In reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree, the tensor a_{ij} has the form*

$$\begin{aligned} a_{il} = g_{il}\frac{a-1}{n-2} - \frac{1}{(n-2)u_\alpha u^\alpha}((a - \frac{n}{2})(u_l u_i + u_{\bar{l}} u_{\bar{i}}) \\ + (1 - \frac{n}{2})(u_{\bar{i}} u_l + u_i u_{\bar{l}})). \end{aligned}$$

Contracting this expression for the tensor a_{il} with a^l_j, taking into account the idempotency of the tensor a_{ij}, and performing an index substitution $j \to l$, we obtain:

$$a_{il} = \frac{1}{2u_\alpha u^\alpha}(u_i + u_{\bar{i}})(u_l + u_{\bar{l}}). \tag{5.22}$$

Multiplying and dividing this expression by $\overset{4}{\tau}$, covariantly differentiating with respect to x^j and taking into account the covariant constancy of a_{ij}, we obtain that the expression is covariantly constant:

$$(\overset{4}{\tau}(u_i + u_{\bar{i}})(u_l + u_{\bar{l}}))_{,j} = 0. \tag{5.23}$$

and, therefore,

$$A_{ij,k} = 0.$$

Substituting (5.22) into (5.17) and using (5.20), we obtain:

$$A_{ij} = (\frac{1}{\overset{4}{\tau} u_\alpha u^\alpha} - \overset{4}{\tau} u_\alpha u^\alpha)a_{ij}. \tag{5.24}$$

Conjugating this expression over all indices and subtracting it from (5.24), we get:

$$(\frac{1}{\overset{4}{\tau} u_\alpha u^\alpha} - \overset{4}{\tau} u_\alpha u^\alpha)(a_{ij} - a_{\bar{i}\bar{j}}) = 0.$$

Here, one of the factors is zero, and if it is the second factor, i.e., $a_{ij} = a_{\bar{i}\bar{j}}$, then substituting this into (5.21) we find that $u_{\bar{i}} u_{\bar{j}} = u_i u_j$, which is impossible. Therefore,

$$\frac{1}{\overset{4}{\tau} u_\alpha u^\alpha} - \overset{4}{\tau} u_\alpha u^\alpha = 0,$$

that is,

$$\overset{4}{\tau} u_\alpha u^\alpha = \hat{\tau}, \quad \hat{\tau} := \pm 1 \tag{5.25}$$

or, substituting (5.20):

$$\overset{3}{\tau} = \frac{\hat{\tau}}{2}. \tag{5.26}$$

Formulas (5.25) and (5.26) indicate the constancy of the scalar curvature from identity (4.8):

$$R = \hat{\tau}(n^2 - 8).$$

Note that the quadratic equation $n^2 = 8$ has no solutions in the set of natural numbers, therefore it is proven:

Corollary 5.8. *Reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree have constant, non-zero curvature.*

Covariantly differentiating the Ricci tensor from formula (4.7) with respect to x^k, we obtain:

$$R_{ij,k} = (n-4)(\overset{4}{\tau}(u_i u_j + u_{\bar{i}} u_{\bar{j}}))_{,k},$$

but from (5.23) it follows that

$$(\overset{4}{\tau}(u_i u_j + u_{\bar{i}} u_{\bar{j}}))_{,k} = -(\overset{4}{\tau}(u_i u_{\bar{j}} + u_{\bar{i}} u_j))_{,k},$$

therefore, the covariant derivative of the Ricci tensor takes the form

$$R_{ij,k} = -(n-4)(\overset{4}{\tau}(u_i u_{\bar{j}} + u_{\bar{i}} u_j))_{,k}.$$

In view of (5.21), it is proven:

Corollary 5.9. *In reducible spaces of constant almost holomorphic curvature of type II with an elliptic antisymmetric affinor structure of the second degree, the covariant derivative of the Ricci tensor has the form*

$$R_{ij,k} = \hat{\tau}(n-4)(a_{\bar{i}\bar{j}})_{,k}.$$

Thus, for the studied spaces, the form of the tensor a_{ij} has been obtained, as well as special additional conditions on the objects of the intrinsic geometry of these spaces, particularly on the scalar curvature.

6. Conclusions

Thus, it has been proven that pseudo-Riemannian spaces of constant almost holomorphic curvature of type I are almost Einstein spaces, and the corresponding vector u_i is isotropic.

For reducible spaces of constant almost holomorphic curvature of type II, it has been proven that the invariant $\hat{u}$ can only be equal to 1, and the vector u_i corresponding to this space is non-isotropic. It has been shown that the scalar curvature of these spaces is constant and non-zero, and the Ricci tensor, as well as the curvature tensor, are semisymmetric.

In general, for pseudo-Riemannian spaces of constant almost holomorphic curvature of both types, it has been proven that they are Bochner-flat spaces which are distinct from Einstein spaces.

References

[1] M. Aldi, D. Grandini. Polynomial structures in generalized geometry. *Differential Geometry and its Applications*, 84, 2022. doi:10.48550/arXiv.2106.02798.

[2] D. Barden, C. Thomas. *An introduction to differential manifolds*. University of Cambridge, 2003. doi:10.1142/p285.

[3] D. Doikov, V. Kiosak. On the Schwarzschild model for gravitating objects of the Universe. *AIP Conference Proceedings*, 2302:040001, 2020. doi:10.1063/5.0033657.

[4] Yu. Fedchenko, O. Lesechko. Special semi-reducible pseudo-Riemannian spaces. *Proceedings of the International Geometry Center*, 14(1):48–59, 2021. doi:10.15673/tmgc.v14i1.1940.

[5] Yu. Fedchenko, O. Lesechko, Ya. Marianko. Semi-reducible pseudo-Riemannian spaces with additional conditions. *Proceedings of the International Geometry Center*, 18(2):233–247, 2025. doi:10.15673/pigc.v18i2.3035.

[6] R. L. Fernandes. *Lectures on differential geometry*. World Scientific, 2014. doi:10.1142/12733.

[7] S. Gallot, D. Hulin, J. Lafontaine. *Riemannian geometry*. Universitext. Springer-Verlag, Berlin, 3 edition, 2004. doi:10.1007/978-3-642-18855-8.

[8] C. J. Isham. *Modern differential geometry for physicists*. World Scientific, 2 edition, 1999. doi:10.1142/3867.

[9] V. Kiosak, L. Kusik, V. Isaiev. Geodesic Ricci-symmetric pseudo-Riemannian spaces. *Proceedings of the International Geometry Center*, 15(2):110–120, 2022. doi:10.15673/tmgc.v15i2.2224.

[10] V. Kiosak, O. Latysh, V. Kuzmich. Split curvature. *Proceedings of the International Geometry Center*, 17(3):232–243, 2024. doi:10.15673/pigc.v17i3.2817.

[11] V. Kiosak, O. Lesechko, O. Latysh. On geodesic mappings of symmetric pairs. *Proceedings of the International Geometry Center*, 15:230–238, 2022. doi:10.15673/tmgc.v15i3-4.2430.

[12] V. Kiosak, O. Lesechko, O. Savchenko. Mappings of spaces with affine connection. In *17th Conference on Applied Mathematics, APLIMAT*, pages 563–569, Bratislava, 2018.

[13] V. Kiosak, V. S. Matveev. There exist no 4-dimensional geodesically equivalent metrics with the same stress-energy tensor. *Journal of Geometry and Physics*, 78:1–11, 2014. doi:10.1016/j.geomphys.2014.01.002.

[14] V. Kiosak, O. Prishlyak, O. Gudyreva. On geodesic mappings of threesymmetric spaces. *Proceedings of the International Geometry Center*, 17(1):56–64, 2024. doi:10.15673/pigc.v17i1.2647.

[15] V. Kiosak, O. Prishlyak, O. Lesechko. On the geodesic mappings of pseudo-Riemannian spaces with special supplementary tensor. *Proceedings of the International Geometry Center*, 14(4):13–26, 2021. doi:10.15673/tmgc.v14i4.2140.

[16] V. Kiosak, A. Savchenko, O. Gudyreva. On the conformal mappings of special quasi-Einstein spaces. *AIP Conference Proceedings*, 2164:040001, 2019. doi:10.1063/1.5130793.

[17] V. Kiosak, A. Savchenko, A. Kamienieva. Geodesic mappings of compact quasi-Einstein spaces with constant scalar curvature. *AIP Conference Proceedings*, 2302:040002, 2020. doi:10.1063/5.0033661.

[18] V. Kiosak, A. Savchenko, S. Khniunin. On the typology of quasi-Einstein spaces. *AIP Conference Proceedings*, 2302:040003, 2020. doi:10.1063/5.0033700.

[19] V. Kiosak, A. Savchenko, G. Kovalova. Geodesic mappings of compact quasi-Einstein spaces, i. *Proceedings of the International Geometry Center*, 13(1):35–48, 2020. doi:10.15673/tmgc.v13i1.1711.

[20] V. Kiosak, A. Savchenko, L. Kusik. On the properties of Ricci solitons. *AIP Conference Proceedings*, 2522:120006, 2022. doi:10.1063/5.0100792.

[21] V. Kiosak, A. Savchenko, O. Latysh. Geodesic mappings of compact quasi-Einstein spaces, ii. *Proceedings of the International Geometry Center*, 14(1):80–91, 2021. doi:10.15673/tmgc.v14i1.1936.

[22] V. A. Kiosak, G. V. Kovalova. Geodesic mappings of quasi-Einstein spaces with a constant scalar curvature. *Mathematical Studies*, 53(2):212–217, 2020. doi:10.30970/ms.53.2.212-217.

[23] O. Lesechko, O. Latysh, T. Spychak. Conformally flat Kähler spaces. In *AIP Conference Proceedings*, volume 2302, page 040004, 2020. doi:10.1063/5.0034024.

[24] O. Lesechko, A. Soloviov. Conformally flat Kähler spaces. *Researches in Mathematics and Mechanics*, 28(1–2 (41–42)):47–63, 2023. doi:10.18524/2519-206x.2023.1-2(41-42).305257.

[25] W. Rong, C. Yue. *An Introduction to differential geometry and topology in mathematical physics.* Zhejiang University, PR China, 1999. doi:10.1142/3871.

[26] A. Savchenko, T. Shevchenko, S. Hedulian. Conformal recurrent Kähler spaces. *Proceedings of the International Geometry Center*, 17(1):88–98, 2024. doi:10.15673/pigc.v17i1.2752.

[27] A. Savchenko, N. Vashpanova, N. Vasylieva. Generalized ϕ(Ric)-vector fields in special pseudo-Riemannian spaces. *Proceedings of the International Geometry Center*, 14(4):1–12, 2021. doi:10.15673/tmgc.v14i4.2155.

[28] A. Savchenko, M. Zarichnyi. Metrization of free groups on ultrametric spaces. *Topology and its Applications*, 157(4):724–729, 2010. doi:10.1016/j.topol.2009.08.015.

[29] G. J. Sussman, J. Wisdom, W. Farr. *Functional differential geometry.* MIT Press, Cambridge, Massachusetts and London, England, 2013. Available at `https://groups.csail.mit.edu/mac/users/gjs/6946/calculus-indexed.pdf`.

[30] C. H. Taubes. *Differential geometry: bundles, connections, metrics and curvature*, volume 23 of *Oxford Graduate Texts in Mathematics.* Oxford University Press, 2011. doi:10.1093/acprof:oso/9780199605880.001.0001.

[31] M. Zarichnyi, A. Savchenko, V. Kiosak. Strong topology on the set of persistence diagrams. *AIP Conference Proceedings*, 2164:040006, 2019. doi:10.1063/1.5130798.

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