

International Conference
Morse theory and its applications
dedicated to the memory and 70th anniversary of
Volodymyr Sharko (25.09.1949-07.10.2014)



On September 25-28, 2019 Institute of Mathematics of NAS of Ukraine organized an International conference «**Morse theory and its applications**» dedicated to the memory and 70th anniversary of Volodymyr Sharko. The conference held at the National Dragomanov Pedagogical University in Kyiv.

The topic of the conference was closely related with the mathematical interests of Volodymyr Sharko. There was nearly one hundred participants from Australia, Brasil, China, Denmark, Georgia, Germany, France, Japan, Iran, Poland, Portugal, Russia, Senegal, Singapore, Slovenia, United Arab Emirates, United Kingdom, Ukraine, USA. Among them were students and students of students of V. Sharko, his friends and people who know him personally. Alexey Bolsinov (UK), Dmitri Millionshchikov (Russia), and Oleg Musin (USA) were not able to come and they recorded video appeals to participants with their memories of Volodymyr Sharko.

Morse theory arose from a very general observation that the number and types of critical points of a smooth function on a manifold reflects the topological properties of that manifold. This observation has significant applications and generalizations in many areas of mathematic, physics and other parts of sciences which use geometric models. Therefore a lot of different topics were touched at this conference.

For instance, there were talks by participants from Ukraine: Taras Banakh, Sergey Chuiko, Kaveh Eftekharinasab, Bohdan Feshchenko, Nikolaj Glazunov, Oleg Gutik, Svitlana Halushchak, Hossein Hatamian, Olena Karlova, Olexandra

Khokhliuk, Arkadii Kindybaliuk, Anna Kravchenko, Iryna Kuznietsova, Sergiy Maksymenko, Tetiana Obikhod, Eugene Polulyakh, Alexandr Prishlyak, Anatolij Prykarpatski, Ruslan Skuratovskii, Yuliia Soroka, Taras Vasylyshyn, Igor Vlasenko, Liudmyla Vyhivska, Iryna Yurchuk, Andriy Zagorodnyuk, Myhailo Zarithnyi, from Poland: Marek Golasiński, Mariusz Kwiatkowski, Łukasz Michalak, Mark Pankov, Leonid Plachta, Juliusz Stochmal, Marek Wójtowicz, from Japan: Kaoru Ono, Jun Ueki, Dominik Wrazidlo, from Russia: Ivan Dynnikov, Taras Panov, Arkadiy Skopenkov, as well as by Zhi Lü (China), Søren Galatius (Denmark), Vladimir Vershinin (France), Katherina von Dichter (Germany), Jose Oliveira (Portugal), Jie Wu (Singapore), Ilya Spitkovsky (United Arab Emirates).

Video records of the talks can be found at the conference web-page

<https://www.imath.kiev.ua/~topology/conf/shatrko70>

The conference started exactly at the 70th birthday of Volodymyr Sharko who was born on September 25, 1949 in the town Otynia of Ivano-Frankivsk region. His father Vasyl Ivanovych was a teacher in mathematics in Izyum town.



Volodymyr Sharko and his parents:
Vasyl Ivanovych and Kateryna Pylypivna.



Volodymyr Sharko

In 1959 Volodymyr Sharko entered Ivano-Frankivsk school no. 5 (which is now a specialized school of I-III degrees with advanced study of German language) and finished it in 1966 at the age of 17 years.

After school Volodymyr Sharko entered Stanislav State Medical University (now Ivano-Frankivsk National Medical University). However (as he told himself) it turned out psychically hard for him to visit a morgue for learning human bodies, and he left that university and worked in 1966-1968 as a locksmith at Carpathian furniture combine.

Further, in 1968 he entered Kyiv State University to the Faculty of Mechanics and Mathematics and finished it in 1973 with the speciality «Mathematics». Then he became a postgraduate student of Institute of Mathematics of Ukrainian Academy of Sciences in Kyiv. His scientific adviser was Yuri Yuriyovych Trokhimchuk. From that time Volodymyr Sharko was working at the Institute of Mathematics all his life.

At the beginning of 1960s Yu. Mitropolskiy (director of the Institute of Mathematics) proposed an idea to organize mathematical summer schools, similar to summer schools that held at that time in Italy and France. That proposition were supported by Academy of Sciences and it was decided that such schools will be organized by Institute of Mathematics of Ukrainian Academy of Sciences and Steklov Mathematical Institute. The First school started in 1963 in Kaniv (Ukraine), from II till X in Katsiveli (Crimea), XI in Kolomyya (Ivano-Frankivsk region), XII in Kyiv, and XII in Katsiveli.



Joint photo of several staff members of the Institute of Mathematics

Yu. Trokhimchuk, the scientific adviser of Volodymyr Sharko, organized those summer schools together with his postgraduate students. Many prominent mathematicians gave lectures at those summer schools. Volodymyr Sharko met there Mikhail Postnikov and his students Anatoliy Fomenko, Alexander Mishchenko and many Moscow geometers and topologists and probably due to them he brought the ideas of differential topology and Morse theory to Kiyv.



Yu. Trokhimchuk and V. Sharko at a conference lecture

The first paper of Volodymyr Sharko was a preprint in which he extended S. Smale's results about existence of minimal Morse functions on simply connected

manifolds to PL and TOP categories, and for high dimensional manifolds $\dim \geq 6$ with $\pi_1 = \mathbb{Z}^n$. Further he found certain invariants of epimorphisms $\mathbb{Z}^n \rightarrow G$, where G is an abelian group having n generators. This allowed him to classify exact Morse functions on manifolds with $\dim \geq 6$ and describe path components of the spaces of such functions. These results were rethought in a series of papers, and finally were collected in the book «Functions on manifolds» published in 1991 in Russian and then translated to English in 1993 by American Mathematical Society.

He often visit seminars in Moscow State University organized by Mikhail Postnikov and gave many talks on them and had many friends there. Volodymyr Sharko defended

- Dissertation of Candidate of sciences «**On exact Morse functions**» in 1976 at the Institute of Mathematics of NAS of Ukraine (PhD), and
- Dissertation of Doctor of sciences «**Minimal Morse functions**» in 1987 at Steklov Mathematical Institute. One of his opponents was A. Fomenko.



Members of Topology department (2012).

Upper row from the left: S. Maksymenko, V. Sergeichuk, Yu. Sharko, V. Sharko, I. Vlasenko, O. Prishlyak. Lower row: Ye. Polulyakh, V. Krouglov, D. Goltsov.

In 1986 A. Fomenko discovered that the problem of topological classification of «typical» integrable Hamiltonian systems with two degrees of freedom reduces to the study of topological structures of Bott functions on 3-manifolds and Morse functions on surfaces.

A natural task was to look at Bott functions whose critical points are disjoint unions of circles (called *round Morse functions*), and try to construct such functions with minimal possible number of such circles (*exact round Morse functions*). The methods developed by Volodymyr Sharko for exact Morse functions became very useful here. During 1986-1995 V. Sharko wrote several joint papers with A. Fomenko, S. Matveev, A. Bolsinov concerning structures of round Morse functions.

In 1998 A. Oshemkov and V. Sharko fixed a result by Peixoto on classification of Morse-Smale vector fields.

In that year V. Sharko also classified path components of the spaces of Morse functions on surfaces. It was independently proved by Sergey Matveev and Heiner Zieschang.

From 2000 Volodymyr Sharko started to study C^* -algebras and chain complexes over them. He extended his methods for constructing minimal Morse functions and defined L^2 -analogues of his invariant allowing to get exact numbers of closed orbits of vector fields of high dimensional manifolds.

Another part of V. Sharko' activity concerned with the structure of Reeb graphs of smooth functions with isolated critical points of non-compact surfaces, in particular for pseudoharmonic functions. During his life Volodymyr Sharko wrote nearly 100 mathematical papers. Some of them leaved non-finished and his pupils finished and published them later.

The last paper by Volodymyr Sharko dealt with discrete analogues of Lyapunov functions for dynamical systems.

Pupils and PhD students of Volodymyr Sharko and their dissertations

- 1984 ¹**Igor Solopko**, «Singularities of functions and geometry of manifolds»
- 1987 ¹**Eugene Mykhailyuk**, «Morse functions»
- 1990 **Aswad Haysam**, «Morse functions and forms on manifolds»
- 1991 **Yury Shkolnikov**, «Morse functions on one-connected five-dimensional manifolds»
- 1993 **Bondar Olga**, «R-categories and functions with degenerate singular subsets»
- 1993 **Olexandr Pryshlyak**, «Morse differential equations and functions on varieties and pairs of varieties»
- 1994 **Olena Girik**, «Vector fields on two-dimensional varieties»
- 1996 **Dmitry Poltavets**, «Topology of dynamic systems on surfaces»
- 1998 **Mark Pankov**, «Irregular subsets of manifolds and their applications to map theories»
- 1998 **Eugene Pollyakh**, «Some topological properties of Pontryagin's dynamic systems»
- 1999 **Sergiy Maksymenko**, «Morse functions of surfaces»
- 2002 **Igor Vlasenko**, «Dynamic Morse Smale systems with discrete time on surfaces»
- 2003 **Alexandera Mozgova**, «Four-dimensional graph manifolds»
- 2005 **Viktor Urmachev**, «Two-dimensional dynamic systems with impulse action»
- 2006 **Olexandr Kadubovsky**, «Vector fields and Lyapunov functions on surfaces»
- 2007 **Olena Andriyuk**, «Functions on one-dimensional varieties»
- 2008 **Irina Yurchuk**, «Topological equivalence of pseudoharmonic functions»
- 2009 **Tetyana Budnytska**, «Topological conjugation of functions»

- 2011 **Kaveh Eftekharinasab**, «Gauss curvature and Frechet manifolds»
 2018 ²**Yuliya Soroka**, «Automorphisms of foliations on two-dimensional non-compact surfaces»

Doctorants of Volodymyr Sharko

- 2005 **Olexandr Pryshlyak**, «Topological properties of functions and vector fields on low-dimensional manifolds»
 2008 **Leonid Plakhta**, «Invariants of nodes and surfaces in 3-dimensional space»
 2008 **Mark Pankov**, «Transformations of Grassmanians»
 2011 **Sergiy Maksymenko**, «Smooth shifts along the orbits of flows and their application»
 2018 ²**Eugene Polulyakh**, «Topology of singular foliations and related topics»

¹ Formally the adviser was Yu. Trokhimchuk, since V. Sharko was not a Doctor of sciences at that time.

² Defended dissertation after V. Sharko has passed away.

Career of Volodymyr Sharko

- Department of geometric function theory (1976-1987) (young - 1976 - PhD, senior - 1980, advanced researcher - 1986);
- Department of algebra and geometric methods of analysis (1987-1996) (advanced researcher);
- Department of approximation theory (1996-2001) (advanced researcher);
- Department of topology (2001-2008) (Head of department);
- Deputy director for science (2008-2014).

Volodymyr Sharko was also a

- Deputy of Academician-Secretary of Mathematical Section of the National Academy of Sciences of Ukraine;
- Deputy Editor-in-Chief of Ukrainian Mathematical Journal;
- member of Editorial board of journals *Methods of Functional Analysis*, *Mathematical Bulletin of the Shevchenko Scientific Society*;
- Editor in chief of «Proceedings of the International Geometry Center».

Volodymyr Sharko was also awarded by «Mykola Ostrovskiy Prize» (1980), «Krylov Prize» (2005), «State prize of Ukraine in Science and Technology» (2006), and «Lavrentyev Prizes of the National Academy of Sciences of Ukraine» (2010). In 2006 V. Sharko was elected a **Corresponding Member** of the National Academy of Sciences of Ukraine.

People know Volodymyr Sharko as very kind, polite and attentive to others.

The current volume of «Proceedings of the International Geometry Center» contains papers devoted to Morse theory and dedicated to the memory and 70th anniversary of Volodymyr Sharko.

Olexandr Prishlyak
 Sergiy Maksymenko
 Nadiia Konovenko