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Infinite-dimensional manifolds related to C-spaces

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Abstract. Haver's property C turns out to be related to Borst's transfinite extension of the covering dimension. We prove that, for a uncountably many countable ordinals β there exists a strongly universal k_ω-space for the class of spaces of transfinite covering dimension $< \beta$. In some sense, our result is a k_ω-counterpart of Radul's theorem on existence of absorbing sets of given transfinite covering dimension.

Анотація. Як з'ясувалося, властивість C Гейвера споріднена з трансфінітним розширенням Борста покриттєвого виміру. Ми доводимо, що для нескінченного числа зліченних ординалів β існує сильно універсальний k_ω-простір для класу просторів трансфінітного покриттєвого виміру $< \beta$. В деякому сенсі наш результат є k_ω-аналогом теореми Радула про існування поглинаючих множин заданого трансфінітного покриттєвого виміру.

1. INTRODUCTION

The notion of metric C-space was introduced by W. Haver [9]. He applied these spaces to the theory of retracts. In [1] the property C was defined for all topological spaces. The C-spaces play an important role in the dimension theory.

P. Borst [7] introduced a transfinite extension of the covering dimension dim which characterizes property C.

T. Radul [13] proved that there exists an uncountable set of countable ordinals β such that there exist noncountable-dimensional pre-Hilbert spaces

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D_β which are absorbing spaces (in the sense of Bestvina and Mogilski [6]) for the class of compacta with $\dim_C$ less than β.

In some sense, our main result is a counterpart of Radul's theorem in the category of k_ω-spaces. We prove that, for an uncountable set of countable ordinals β, there exists a k_ω-space which is strongly universal for the class of compacta with $\dim_C$ less than β.

2. PRELIMINARIES

A family $\mathcal{V}$ of subsets of a space X is said to *refine* a family $\mathcal{U}$ of subsets of X if for each element $V \in \mathcal{V}$ there exists $U \in \mathcal{U}$ for which $V \subset U$. A family $\mathcal{V}$ is said to *star-refine* a family $\mathcal{U}$ if for every $U \in \mathcal{U}$ there exists $V \in \mathcal{V}$ such that $U' \subset V$ for any $U' \in \mathcal{U}$ such that $U \cap U' \neq \emptyset$.

A family $\mathcal{V}$ of subsets of X is called *disjoint* if every two elements of $\mathcal{V}$ are disjoint and is open if each element of $\mathcal{V}$ is open. The family of all open coverings of a space X is denoted by $\mathrm{cov}(X)$.

If $\mathcal{U}$ is a family of subsets in a metric space, then we define

$$\mathrm{mesh}(\mathcal{U}) = \sup\{\mathrm{diam}(U) \mid U \in \mathcal{U}\}$$

(as usual, $\mathrm{diam}(U)$ is the diameter of U).

Definition 2.1. A space X has *property C* (briefly is a *C-space*) if for each sequence $\{\alpha_n \mid n \in \mathbb{N}\}$ of open coverings of X there exists a sequence $\{\beta_n \mid n \in \mathbb{N}\}$ of open disjoint families such that each family β_n refines α_n and $\cup_{n=1}^{\infty}\beta_n \in \mathrm{cov}(X)$.

We will need the following properties of compact metrizable C-spaces (see [1]), where they are proved for more general class of spaces).

Proposition 2.2. *Every closed subspace of a C-space is a C-space.*

Proposition 2.3. *If $X = \cup_{n=1}^{\infty}X_n$, where X_n is a C space for any $n \in \mathbb{N}$, then X is a C-space.*

Corollary 2.4. *Let A be a closed subset of a compact metrizable C-space X. Then the quotient space X/A is a C-space.*

Proof. Let $\{U_n \mid n \in \mathbb{N}\}$ be a countable family of neighborhoods of A such that $A \cap_{n=1}^{\infty} U_n$. Then X/A is the sum of the sets homeomorphic to $X \backslash U_n$ and a singleton. □

The following statement is proved in [8] for the paracompact spaces.

Proposition 2.5. *Let f be a closed map from a compact metrizable space X onto a C-space Y. If $f^{-1}(y)$ has property C for each $y \in Y$, then X is a C-space.*

We will need the following result by T. Radul [13].

Theorem 2.6. *For each $\alpha < \omega_1$ there exists a compact metrizable C-space L_α which contains topologically each compact metrizable space K with* $\dim_C K \leqslant \alpha$.

2.7. **Dimension** $\dim_C$. P. Borst [7] introduced the transfinite extension $\dim_C$ of the covering dimension. We recall some necessary definition. Let us start with the ordinal number Ord.

Given a set L, we denote by $\operatorname{Fin} L$ the collection of all finite, nonempty subsets of L. Let M be a subset of $\operatorname{Fin} L$. For $\sigma \in \{\varnothing\} \cup \operatorname{Fin} L$, put

$$M^\sigma = \{\tau \in \operatorname{Fin} L \mid \sigma \cup \tau \in M \text{ and } \sigma \cap \tau = \varnothing\}.$$

We write M^a instead of $M^{\{a\}}$.

Define the ordinal number $\operatorname{Ord} M$ inductively as follows:

(1) $\operatorname{Ord} M = 0$ if and only if $M = \varnothing$,
(2) $\operatorname{Ord} M \leqslant \alpha$ if and only if for every $aE \in L$, $\operatorname{Ord} M^a < \alpha$,
(3) $\operatorname{Ord} M = \alpha$ if and only if $\operatorname{Ord} M \leqslant \alpha$ and $\operatorname{Ord} M < \alpha$ is not true,
(4) $\operatorname{Ord} M = \infty$ if and only if $\operatorname{Ord} M > \alpha$ for every ordinal number α.

Let X be a topological space and $K(X)$ denote the set of the all locally finite coverings of X. Put

$$M_{K(X)} = \Big\{ \{\alpha_i\}_{i=1}^n \in \operatorname{Fin} K(X) \ \mid \text{ there are no open disjoint families } \beta_i, i = 1, \dots, n, \text{ such that } \beta_i \text{ refines } \alpha_i \text{ and } \cup_{i=1}^n \beta_i \text{ covers } X, n \in \mathbb{N} \Big\}.$$

Definition 2.8. For any topological space X we set

$$\dim_C X = \operatorname{Ord} M_{K(X)}.$$

Remark that the dimension $\dim_C$ coincides with classical covering dimension dim for finite-dimensional spaces [7] and for any compact metric space K $\dim_C K$ exists if and only if K has property C.

Also, it is an easy consequence of the definition that if A is a closed subset of X, then $\dim_C A \leqslant \dim_C X$.

Let us denote by $\mathcal{D}(\beta)$ the class of compact metric spaces with $\dim_C$ less than β. We say that a topological space Y is *$\mathcal{D}(\beta)$-universal* if Y contains topologically all compacta from $\mathcal{D}(\beta)$.

T. Radul [13, Theorem 3] proved that for each ordinal $\alpha < \omega_1$ there exists an ordinal β, $\alpha \leqslant \beta < \omega_1$, and a C-compact metric space X such that $\dim_C X = \beta$ and X is $\mathcal{D}(\beta)$-universal.

Lemma 2.9. *There exists a function* $h\colon \omega_1 \to \omega_1$ *such that, for any compact metric* C*-space* X *and any closed subset* A *of* X*,*

$$\dim_C(X/A) \leqslant h(\dim_C X).$$

Proof. Let K be a universal space for compact metric spaces X with $\dim_C X \leqslant \alpha$. Let $\exp K$ denote the hyperspace of K, i.e., the space of all nonempty closed subsets of X endowed with the Vietoris topology (see, e.g., [11]). There exists a continuous map of the Cantor discontinuum C onto $\exp X$.

Let $Z = C \times K$ and $B = \{(c, x) \mid x \in f(c)\} \subset C \times K$. Clearly, B is a closed subset of Z. Since C is zero-dimensional, Z is a C-space. We let $h(\alpha) = \dim_C(Z/B)$.

Now, suppose that Y is a compact metrizable space and $\dim_C Y \leqslant \alpha$. We may assume that $Y \subset K$. If F is a nonempty closed subset of Y, then $F \in \exp Y \subset \exp K$ and there exists $c \in C$ such that $f(c) = F$. Then Y/F is, clearly, homeomorphic to $(\{c\} \times Y)/(\{c\} \times F)$, and therefore is homeomorphic to a subset of Z/B. We conclude that

$$\dim_C(Y/F) \leqslant \dim_C(Z/B) \leqslant h(\alpha). \qquad \square$$

Remark 2.10. Actually, no example is known witnessing that h is not the identity map.

3. RESULTS

Recall that an *absolute retract* (*AR-space*) is a space X which is a retract of every metric space containing X as a closed subset.

Proposition 3.1. *Let* X *be a compact metrizable* C*-space. Then there exists a compact metrizable* C*-space* $\hat{X}$ *that contains a topological copy of* X *and is an AR-space.*

Proof. We assume that X is a metric space. Define inductively a sequence $(\mathcal{U}_i)$ of open covers of X as follows. Let $\mathcal{U}_1 = \{X\}$. If $\mathcal{U}_j$ is already defined for $j < i$, let $\mathcal{U}_i$ be a cover of X which star-refines $\mathcal{U}_{i-1}$ and such that $\operatorname{mesh}(\mathcal{U}_{i-1}) \leqslant 2^{-i}$.

Assume that X is isometrically embedded into a Banach space L which, in turn, is identified with the subset $L \times \{0\}$ of $L \times [0, 1]$. Let $N(\mathcal{U}_i)$ be the nerve of the cover $\mathcal{U}_i$. Assume also that $N(\mathcal{U}_i)$ is a subpolyhedron of $L \times \{2^{1-i}\}$ with the following property: for any $U \in \mathcal{U}_i$, the vertex of $N(\mathcal{U}_i)$

corresponding to U is an element of the set $U \times \{2^{1-i}\}$. For every natural i, let K_i be the mapping cylinder of a natural map $f_i\colon N(\mathcal{U}_{i+1}) \to N(\mathcal{U}_i)$. The latter is a simplicial map sending the vertex corresponding to $U \in N(\mathcal{U}_{i+1})$ to the vertex corresponding to any $V \in N(\mathcal{U}_i)$ such that $U \subset V$. We assume as well that the mapping cylinder consists of all linear segments in $L \times [2^i, 2^{i-1}]$.

Finally, let $\hat{L} = L \bigcup \cup_{i=1}^{\infty} K_i$. Using standard arguments we show that $\hat{L}$ is a compact metrizable AR-space. The projection of $\hat{L}$ onto $[0,1]$ is a closed map such that every its preimage is a C-space (either L or a polyhedron). Therefore, $\hat{L}$ is a C-space. ☐

Proposition 3.2. *For any $\alpha < \omega_1$ there exists a pointed compact metrizable C-space $(\tilde{L}, *)$ that contains a topological copy of each pointed compact metrizable C-space $(K, *)$ with $\dim_C K \leqslant \alpha$.*

Proof. Let L_α be a universal space for compact metrizable C-spaces K with $\dim_C K \leqslant \alpha$. Denote by $\tilde{L}$ the quotient space $(L_\alpha \times L_\alpha)/\Delta$, where Δ is the diagonal $\{(x,x) \mid x \in L_\alpha\} \subset L_\alpha \times L_\alpha$. The set Δ is regarded as the base point of $\tilde{L}$. Denote by $q\colon L_\alpha \times L_\alpha \to \tilde{L}$ the quotient map.

Suppose that (K, x_0) is a compact metrizable C-space with $\dim_C K \leqslant \alpha$. Then, clearly, the map $f\colon K \to \tilde{L}$ defined by the formula $f(x) = q(x, x_0)$, $x \in K$, is a pointed embedding.

Finally, remark that $\tilde{L}$ is a C-space. Indeed, one can represent $\tilde{L}$ as the countable union of the singleton $\{q(\Delta)\}$ and the spaces $q((L \times L)\backslash U_i)$, where $\{U_i \mid i \in \mathbb{N}\}$ is a countable base neighborhoods of Δ in the product $L \times L$, and then apply Propositions 2.2 and 2.3. ☐

Recall that a topological space X is said to be a *k_ω-space* if $X = \varinjlim X_i$, where (X_i) is an increasing sequence of its compact subspaces.

We say that a space X is *strongly $\mathcal{D}(\beta)$-universal* (resp. *locally strongly $\mathcal{D}(\beta)$-universal*) if for every compact metric space A with $\dim_C(A) < \beta$ and every embedding $f\colon B \to X$ of its closed subset B into X there exists an embedding $\bar{f}\colon A \to X$ (resp. an embedding $\bar{f}\colon U \to X$, where U is a neighborhood of B in A) that extends f.

Theorem 3.3. *There is an uncountable set $\Phi \subset \omega_1$ such that, for every $\beta \in \Phi$ there exists a strongly $\mathcal{D}(\beta)$-universal k_ω-space K_β which is the countable direct limit of an increasing sequence of compact spaces from the class $\mathcal{D}(\beta)$.*

Proof. Let $\alpha < \omega_1$. Let $X_1 = \{*\}$ be any compact metrizable C space with $\dim_C X_1 = \alpha$. Suppose that compact metrizable C-spaces X_i are already constructed for all $i < n$.

By Proposition 3.1, there exists a compact metric C-space $\hat{X}_{n-1}$ which contains X_{n-1} and is an absolute retract.

By Proposition 3.2, there exists a pointed compact C-space $(Y_{n-1}, *)$ which is universal for pointed compact metric spaces of $\dim_C$ not exceeding $h(\alpha)$. Take $X_n = \hat{X}_{n-1} \times Y_{n-1}$.

Let $X = \varinjlim X_n$ and let $\beta = \sup\{\dim_C(X_n) + 1 \mid n \in \mathbb{N}\}$. Then clearly $\beta \geqslant \alpha$.

We are going to show that X is strongly $\mathcal{D}(\beta)$-universal. Suppose that (A, B) is a pair of compact metric spaces and $\dim_C A < \beta$. Suppose also that $f\colon B \to X$ is an embedding. Since X is a k_ω-space, there exists $n \in \mathbb{N}$ such that $f(B) \subset X_n$. Since $\hat{X}_n$ is an AR-space, there is a continuous extension $f'\colon A \to \hat{X}_n$.

Let $q\colon A \to A/B$ be the quotient map. By Proposition 3.2, there exists an embedding $g\colon A/B \to Y_n$ such that $g(B) = *$.

Finally, define $\bar{f}\colon A \to X_{n+1}$ by the formula $\bar{f}(x) = (f'(x), gq(x))$. It is easy to see that $\bar{f}$ is an embedding that extends f. $\square$

The following is a characterization theorem for the spaces K_β.

Theorem 3.4. *Let X be a k_ω-space and X is countable direct limit of compact metric spaces from the class $\mathcal{D}(\beta)$. Then the following properties are equivalent:*

1. *X is strongly $\mathcal{D}(\beta)$-universal;*
2. *X is homeomorphic to K_β.*

Proof. We apply the back and forth argument used in [14] as well as in another publications. For the sake of reader's convenience, we provide some details.

Let $Y = \varinjlim Y_n$, where $Y_1 \subset Y_2 \subset \dots$ is a sequence of compact spaces such that $Y_n \in \mathcal{D}(\beta)$ for every $n \in \mathbb{N}$.

Let $m_1 = 1$. There exists an embedding $f_1\colon Y_{m_1} \to K_\beta$. Since Y_{m_1} is compact, there is $n_1 \in \mathbb{N}$ such that $f_1(Y_{m_1}) \subset X_{n_1}$. Moreover, since Y is strongly $\mathcal{D}(\beta)$-universal, there exists an embedding $g_1\colon X_{n_1} \to Y$ such that $g_1|f_1(Y_1) = f_1^{-1}$. Then by compactness of X_{n_1}, there exists $m_2 > m_1$ such that $g_1(X_{n_1}) \subset Y_{m_2}$.

Continuing in this way we obtain a commutative diagram

$$\begin{tikzcd}
Y_{m_1} \arrow[r, hook] \arrow[d, "f_1"'] & Y_{m_2} \arrow[r, hook] \arrow[d, "f_2"'] & Y_{m_3} \arrow[r, hook] \arrow[d, "f_3"'] & \dots \\
X_{n_1} \arrow[r, hook] \arrow[ur, "g_1"] & X_{n_2} \arrow[r, hook] \arrow[ur, "g_2"] & X_{n_3} \arrow[r, hook] \arrow[ur, "g_3"] & \dots,
\end{tikzcd}$$

in which $m_1 < m_2 < \dots$, $n_1 < n_2 < \dots$, f_j, g_j are embeddings, $j \in \mathbb{N}$.

Then

$$\begin{aligned} Y &\simeq \varinjlim_m Y_m = \varinjlim_j Y_{m_j} = \varinjlim\{\, Y_{m_1} \xrightarrow{f_1} X_{n_1} \xrightarrow{g_1} Y_{m_2} \xrightarrow{f_2} X_{n_2} \xrightarrow{g_2} \cdots \} \\ &= \varinjlim_j X_{n_j} = \varinjlim_n X_n = K(\beta). \end{aligned}$$ □

A K_β*-manifold* is a Hausdorff space which is locally homeomorphic to open subsets in K_β. We will assume that the K_β-manifolds are k_ω-spaces.

The following is a characterization theorem for the K_β-manifolds.

Theorem 3.5. *Let X be a k_ω-space and X is countable direct limit of compact metric spaces from the class $\mathcal{D}(\beta)$. Then the following properties are equivalent:*

(1) *X is locally strongly $\mathcal{D}(\beta)$-universal;*
(2) *X is a K_β-manifold.*

Actually, the proof of this result can be performed along the line of the proof of [14, Theorem 1.3]. And, similarly as in [14], we obtain the following Open Embedding Theorem (see also [10]).

Theorem 3.6. *Any K_β-manifold can be embedded into the space K_β as an open set.*

4. UNIVERSAL MAPS

The following notion is introduced in [15]. Let $\mathcal{K}_{\mathrm{fd}}$ denote the class of metrizable finite-dimensional compacta. A map $f\colon X \to Y$ is called *strongly* $\mathcal{K}_{\mathrm{fd}}$*-universal* if, for any if for every embedding $\alpha\colon B \to X$ of a closed subset B of a space $A \in \mathcal{K}_{\mathrm{fd}}$ and any map $\gamma\colon A \to Y$ with $f\alpha = \gamma|B$ there is an embedding $\bar{\alpha}\colon A \to X$ such that $f\bar{\alpha} = \gamma$ and $\bar{\alpha}|B = \alpha$.

Let $\mathbb{R}^\infty$ be the direct limit of the sequence

$$\mathbb{R} \to \mathbb{R} \times \{0\} \subset \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R} \times \{0\} \subset \cdots$$

Let Q denote the Hilbert cube $[-1,1]^\omega$. By Q^∞ we denote the direct limit of the sequence

$$Q \to Q \times \{0\} \subset Q \times Q \to Q \times Q \times \{0\} \subset \cdots$$

A strongly $\mathcal{K}_{\mathrm{fd}}$-universal map $f\colon \mathbb{R}^\infty \to Q^\infty$ is constructed in [15].

Theorem 4.1. *There is a strongly $\mathcal{K}_{\mathrm{fd}}$-universal map from $\mathbb{R}^\infty$ to K_β.*

Proof. Without loss of generality we may assume that K_β is embedded in Q^∞ as a closed subset. Let $f\colon \mathbb{R}^\infty \to Q^\infty$ be an $\mathcal{K}_{\mathrm{fd}}$-universal map. Define $X = f^{-1}(K_\beta)$ and let f' denote the restriction $f' = f|X\colon X \to K_\beta$. Since

K_β is closed in Q^∞, we see that X is a k_ω-space. Clearly, $X = \varinjlim X_i$, where $X_i \in \mathcal{K}_{\mathrm{fd}}$ for all i. Since K_β is an absolute extensor, the space X satisfies the conditions of the characterization theorem for $\mathbb{R}^\infty$ (see [14]). Also, this implies the strong $\mathcal{K}_{\mathrm{fd}}$-universality of f'. □

Remark also that a strongly $\mathcal{K}_{\mathrm{fd}}$-universal map from Theorem 4.1 is unique up to homeomorphism.

5. Remarks

In connection with Radul's results on existence of $\mathcal{D}(\beta)$-absorbing sets in the sense of Bestvina and Mogilski [6] the following question arises.

Question 5.1. *Are there ordinals $\beta < \omega_1$ for which both D_β and K_β exist? Is there a bitopological characterization of this pair in the spirit of Banakh and Sakai* [4]*?*

Recall that in [4] a characterization of the bitopological space $(\mathbb{R}^\infty, \ell^2_{\mathrm{f}})$ is given.

We expect the negative answer to the following question related to Theorem 4.1.

Question 5.2. *Is there a strongly $\mathcal{D}(\beta)$-universal map $K(\beta) \to Q^\infty$?*

Also, we conjecture that the universal map from Theorem 4.1 is not locally self-similar. (Here, a map $\pi\colon X \to Y$ is said to be *locally self-similar* if for every point $x \in X$ and every neighborhood $U \subset X$ of x there is a neighborhood $V \subset U$ of x such that the map $\pi|V\colon V \to \pi(V)$ is homeomorphic to π. It is proved in [3] that the universal map $\mathbb{R}^\infty \to Q^\infty$ is not locally self-similar.)

Banakh and Repovš [3] proved that there exists a linear realization of the universal map $\mathbb{R}^\infty \to Q^\infty$. It looks plausible that such a realization can be found for the universal map from Theorem 4.1. This would provide another construction of the spaces K_β, namely as a linear topological space.

Question 5.3. *Are there free topological groups, semigroups, semilattices etc. homeomorphic to K_β?*

See, e.g., [2, 5, 15] for various results concerning infinite-dimensional manifolds in topological algebra.

Finally, remark that some of the results of this note are announced in [12]; here they are given with new proofs.

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