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A metrizable Lawson semitopological semilattice with non-closed partial order

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Abstract. We construct a metrizable Lawson semitopological semilattice X whose partial order $\leqslant_X = \{(x,y) \in X \times X : xy = x\}$ is not closed in $X \times X$. This resolves a problem posed earlier by the authors.

Анотація. Побудовано лоусонівську напівтопологічну напівгратку з незамкненим частковим порядком. Це розв'язує одну (раніше сформульовану авторами) проблему.

In this paper we shall construct an example of a metrizable Lawson semitopological semilattice with non-closed partial order, thus answering a problem posed by the authors in [2].

A *semilattice* is a commutative semigroup X whose any element $x \in X$ is an *idempotent* in the sense that $xx = x$. An example of a semilattice is any partially ordered set X in which any finite non-empty set $F \subseteq X$ has the greatest lower bound $\inf(F)$. In this case the binary operation $X \times X \to X$, $(x,y) \mapsto \inf\{x,y\}$, turns X into a semilattice.

Each semilattice X carries a partial order $\leqslant$ defined by $x \leqslant y$ iff $xy = x$. For this partial order we have $xy = \inf\{x,y\}$.

A semilattice X is called

- $\uparrow$-*finite* if for every element $x \in X$ its upper set ${\uparrow}x := \{y \in X : x \leqslant y\}$ is finite;
- $\downarrow$-*finite* if for every element $x \in X$ its lower set ${\downarrow}x := \{y \in X : y \leqslant x\}$ is finite.

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A *(semi)topological semilattice* is a semilattice X endowed with a topology such that the binary operation $X \times X \to X$, $(x, y) \mapsto xy$, is (separately) continuous. A semitopological semilattice is called *Lawson* if it has a base of the topology consisting of open subsemilattices.

The continuity of the semilattice operation in a Hausdorff topological semilattice implies the following well-known fact, see [4, VI-1.14].

Proposition 1. For any Hausdorff topological semilattice X the partial order

$$\leqslant_X := \{(x, y) \in X \times X : xy = x\}$$

is a closed subset of $X \times X$.

Also we have the following closedness result, proved in [1, 7.10].

Proposition 2. For any $\downarrow$-finite Hausdorff semitopological semilattice X the partial order $\leqslant_X$ is closed in $X \times X$.

More conditions implying the closedness of the partial order of a Hausdorff semitopological semilattice can be found in [1, §7].

On the other hand, the following example constructed in [2] shows that Proposition 2 does not have conterparts for $\uparrow$-finite semitopological semilattices.

Example 3. There exists a $\uparrow$-finite metrizable semitopological semilattice X whose partial order is dense and non-closed in $X \times X$.

At the end of the paper [2] the authors asked whether there exists a *Lawson* Hausdorff semitopological semilattice X with non-closed partial order. In this paper we shall give an affirmative answer to this question. Moreover, for any cardinal κ we shall construct a Lawson semitopological semilattice X with non-closed partial order such that the topological space of X is a P_κ-space.

A topological space (X, τ) is called a *P_κ-space* if for any family $\mathcal{U} \subseteq \tau$ of cardinality $|\mathcal{U}| \leqslant \kappa$ the intersection $\bigcap \mathcal{U}$ belongs to the topology τ. Each topological space is a P_κ-space for any finite cardinal κ.

A topological space X is called *zero-dimensional* if it has a base of the topology, consisting of clopen sets. A subset of a topological space is *clopen* if it is both closed and open. It is easy to see that every regular P_ω-space is zero-dimensional. The *weight* of a topological space (X, τ) is the smallest cardinality of a base of the topology τ.

The following example (answering [2, Problem 1]) is the main result of this paper.

Example 4. For any infinite cardinal λ there exists a Lawson semitopological semilattice X having the following properties:

(1) the partial order $\{(x,y)\in X\times X : xy=x\}$ of X is not closed in $X\times X$;
(2) the semilattice X is $\uparrow$-finite;
(3) the cardinality and the weight of the space X both are equal to λ;
(4) X is a Hausdorff zero-dimensional space;
(5) X is a P_κ-space for any cardinal $\kappa<\mathrm{cf}(\lambda)$;
(6) if $\lambda=\omega$, then the countable space X is metrizable.

Proof. We identify the cardinal λ with the set $[0,\lambda)$ of ordinals, smaller than λ. For two ordinals $\alpha<\beta$ in λ by $[\alpha,\beta)$ we denote the order interval consisting of ordinals γ such that $\alpha\leqslant\gamma<\beta$.

Consider the semilattice $\{0,1,2\}$, endowed with the operation of minimum. In its power $\{0,1,2\}^\lambda$ consider the subsemilattice X consisting of the functions $x:\lambda\to\{0,1,2\}$ having finite support $\mathrm{supp}(x):=x^{-1}(\{0,1\})$.

For a function $x\in X$ let $\|x\|$ denote the smallest ordinal $\alpha\in\lambda$ such that $\mathrm{supp}(x)\cap[\alpha,\lambda)=\varnothing$. If $\mathrm{supp}(x)\neq\varnothing$, then $\|x\|=\max(\mathrm{supp}(x))+1$ and hence the ordinal $\|x\|-1$ is well-defined and equals $\max(\mathrm{supp}(x))$.

The following property of the semilattice X easily follows from the definition of X.

Claim 5. The semilattice X has cardinality $|X|=\lambda$ and is $\uparrow$-finite. More precisely, for every $x\in X$ its upper set ${\uparrow}x$ has cardinality $|{\uparrow}x|\leqslant 3^{|\mathrm{supp}(x)|}$.

Now we shall define a topology τ on X turning the semilattice X into a Lawson semitopological semilatice with non-closed partial order.

Write the semilattice X as the union $X_0\cup X_1\cup X_2$ of the subsemilattices

$$X_i:=\{x\in X : x(0)=i\} \text{ for } i\in\{0,1,2\}.$$

For every function $x\in X$ and ordinal $\alpha\in[\|x\|,\lambda)$ we shall define a subsemilattice $V_\alpha(x)$ of X as follows.

If $x\in X_2$, then we put $V_\alpha(x):=\{x\}$.

If $x\in X_1$, then we put

$$\begin{aligned}V_\alpha(x):=&\big\{y\in X_1 : y{\restriction}[1,\alpha)=x{\restriction}[1,\alpha) \text{ and } y([\alpha,\lambda))\subseteq\{0,2\}\big\}\cup\\ &\big\{y\in X_2 : y{\restriction}[1,\alpha)=x{\restriction}[1,\alpha),\ y([\alpha,\lambda))\subseteq\{0,2\},\ \text{and } \|y\|>\alpha\big\}.\end{aligned}$$

If $x\in X_0$, then we put

$$\begin{aligned}V_\alpha(x):=&\big\{y\in X_0 : y{\restriction}[1,\alpha)=x{\restriction}[1,\alpha)\big\}\cup\\ &\big\{y\in X_2 : y{\restriction}[1,\alpha)=x{\restriction}[1,\alpha),\ \|y\|>\alpha \text{ and } y(\|y\|-1))=1\big\}.\end{aligned}$$

It is easy to see that $x\in V_\beta(x)\subseteq V_\alpha(x)$ for any $x\in X$ and ordinals $\alpha,\beta\in\lambda$ with $\|x\|\leqslant\alpha\leqslant\beta$.

Endow X with the topology τ consisting of the sets $U\subseteq X$ such that for any $x\in U$ there exists an ordinal $\alpha\in[\|x\|,\lambda)$ such that $V_\alpha(x)\subseteq U$.

Claim 6. For every $x \in X$ and $\alpha \in [\|x\|, \lambda)$, the set $V_\alpha(x)$ is open in the topological space (X, τ).

Proof. Given any element $y \in V_\alpha(x)$, we should find an ordinal $\beta \in [\|y\|, \lambda)$ such that $V_\beta(y) \subseteq V_\alpha(x)$. Choose any ordinal $\beta \in [\alpha, \lambda)$ such that $\beta \geqslant \|y\|$. If $y \in X_2$, then $V_\beta(y) = \{y\} \subseteq V_\alpha(x)$ and we are done. So, we assume that $y \notin X_2$. In this case $x \notin X_2$.

If $x \in X_1$, then $y \in V_\alpha(x) \backslash X_2 \subseteq X_1$ and hence $y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha)$ and $y([\alpha, \lambda)) \subseteq \{0, 2\}$. To prove that $V_\beta(y) \subseteq V_\alpha(x)$, take any $z \in V_\beta(y)$. The definition of $V_\beta(y)$ for $y \in X_1$ ensures that $z \in X_1 \cup X_2$. If $z \in X_1$, then

$$z{\restriction}[1, \beta) = y{\restriction}[1, \beta) \qquad \text{and} \qquad z([\beta, \lambda)) \subseteq \{0, 2\}.$$

Taking into account that $\alpha \leqslant \beta$, we conclude that

$$z{\restriction}[1, \alpha) = y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha)$$

and

$$\begin{aligned} z([\alpha, \lambda)) &= z([\alpha, \beta)) \cup z([\beta, \lambda)) \\ &= y([\alpha, \beta)) \cup z([\beta, \lambda) \subseteq y([\alpha, \lambda)) \cup z([\beta, \lambda)) \subseteq \{0, 2\}, \end{aligned}$$

witnessing that $z \in V_\alpha(x) \cap X_1$.

If $z \in X_2$, then the definition of $V_\beta(y)$ ensures that $z{\restriction}[1, \beta) = y{\restriction}[1, \beta)$, $z([\beta, \lambda)) \subseteq \{0, 2\}$ and $\|z\| > \beta$. Then

$$\begin{gathered} z{\restriction}[1, \alpha) = y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha), \\ z([\alpha, \lambda)) = z([\alpha, \beta)) \cup z([\beta, \lambda)) = y([\alpha, \beta)) \cup z([\beta, \lambda)) \subseteq \{0, 2\} \end{gathered}$$

and $\|z\| > \beta \geqslant \alpha$, witnessing that $z \in V_\alpha(x) \cap X_2$.

Next, assume that $x \in X_0$. In this case $y \in V_\alpha(x) \backslash X_2 \subseteq X_0$ and

$$y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha).$$

To prove that $V_\beta(y) \subseteq V_\alpha(x)$, take any $z \in V_\beta(y)$. The definition of $V_\beta(y)$ ensures that $z \in X_0 \cup X_2$. If $z \in X_0$, then $z{\restriction}[1, \alpha) = y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha)$ and $z \in V_\alpha(x)$ by the definition of $V_\alpha(x)$. If $z \in X_2$, then $z{\restriction}[1, \beta) = y{\restriction}[1, \beta)$, $\|z\| > \beta$ and $z(\|z\| - 1) = 1$. Then

$$z{\restriction}[1, \alpha) = y{\restriction}[1, \alpha) = x{\restriction}[1, \alpha),$$

$\|z\| > \beta \geqslant \alpha$, and $z(\|z\| - 1) = 1$, witnessing that $z \in V_\alpha(x)$. □

Claim 6 implies

Claim 7. The family $\mathcal{B} := \{V_\alpha(x) : x \in X,\ \alpha \in [\|x\|, \lambda)\}$ is a base of the topology τ.

Claim 8. The weight $w(X, \tau)$ of the topological space (X, τ) equals λ.

Proof. Claim 7 implies that $w(X,\tau) \leqslant |\mathcal{B}| \leqslant \lambda$. Taking to account that X_2 is a discrete subspace of (X,τ), we conclude that $w(X,\tau) \geqslant |X_2| = \lambda$. $\square$

The following claim implies that the topological space (X,τ) is zero-dimensional.

Claim 9. For every $x \in X$ and a non-zero ordinal $\alpha \in [\|x\|,\lambda)$ the set $V_\alpha(x)$ is closed in the topological space (X,τ).

Proof. Given any $y \in X\backslash V_\alpha(x)$, we should find an ordinal $\beta \in [\|y\|,\lambda)$ such that $V_\beta(y) \cap V_\alpha(x) = \varnothing$. We claim that the ordinal $\beta = \max\{\alpha, \|y\|\}$ has the desired property.

Six cases are possible.

1) If $y \in X_2$, then

$$V_\alpha(x) \cap V_\beta(y) = V_\alpha(x) \cap \{y\} = \varnothing$$

and we are done.

2) If $y \in X_0 \cup X_1$ and $x \in X_2$, then $V_\alpha(x) \cap V_\beta(y) = \{x\} \cap V_\beta(y) = \varnothing$ as $\|x\| \leqslant \alpha \leqslant \beta$.

3) $x, y \in X_0$. In this case $y \in X_0\backslash V_\alpha(x)$ implies $y{\restriction}[1,\alpha) \neq x{\restriction}[1,\alpha)$ and hence

$$V_\alpha(x) \cap V_\beta(y) \subseteq V_\alpha(x) \cap V_\alpha(y) = \varnothing.$$

4) $x, y \in X_1$. In this case $y \in X_1\backslash V_\alpha(x)$ implies $y{\restriction}[1,\alpha) \neq x{\restriction}[1,\alpha)$ or $y([\alpha,\lambda)) \nsubseteq \{0,2\}$. If $y{\restriction}[1,\alpha) \neq x{\restriction}[1,\alpha)$, then

$$V_\alpha(x) \cap V_\beta(y) \subseteq V_\alpha(x) \cap V_\alpha(y) = \varnothing.$$

If $y{\restriction}[1,\alpha) = x{\restriction}[1,\alpha)$, then $y([\alpha,\lambda)) \nsubseteq \{0,2\}$ and the choice of the ordinal $\beta \geqslant \|y\|$ guarantees that $y([\alpha,\beta)) \nsubseteq \{0,2\}$. Then for any $z \in V_\beta(y)$ we get

$$z([\alpha,\beta)) = y([\alpha,\beta)) \nsubseteq \{0,2\}$$

and hence $z \notin V_\alpha(x)$, which implies $V_\alpha(x) \cap V_\beta(y) = \varnothing$.

5) $x \in X_0$ and $y \in X_1$. In this case for any

$$z \in V_\alpha(x) \cap V_\beta(y),$$

we have $z \in X_2$. The inclusion $z \in X_2 \cap V_\beta(y)$ ensures that $\|z\| > \beta$ and $z([\beta,\lambda)) \subseteq \{0,2\}$. On the other hand, $z \in X_2 \cap V_\alpha(x)$ implies

$$1 = z(\|z\|-1) \in z([\beta,\lambda)) \subseteq \{0,2\},$$

which is a contradiction witnessing that $V_\alpha(x) \cap V_\beta(y) = \varnothing$.

6) $x \in X_1$ and $y \in X_0$. In this case for any $z \in V_\alpha(x) \cap V_\beta(y)$, we have $z \in X_2$. The inclusion $z \in X_2 \cap V_\alpha(x)$ implies $z([\alpha,\lambda)) \subseteq \{0,2\}$ and the inclusion $z \in X_2 \cap V_\beta(y)$ ensures that $\|z\| > \beta$ and

$$1 = z(\|z\|-1) \in z([\beta,\lambda)) \subseteq z([\alpha,\lambda)) \subseteq \{0,2\},$$

which is a contradiction witnessing that $V_\alpha(x) \cap V_\beta(y) = \varnothing$. □

Claim 10. The topological space (X, τ) is Hausdorff.

Proof. Given two distinct points $x, y \in X$, put $\alpha := \max\{\|x\|, \|y\|\}$ and consider four possible cases.

1) If $x \in X_2$, then by Claims 6 and 9, $O_x := \{x\}$ and $O_y := X \setminus \{x\}$ are disjoint clopen neighborhoods of the points x, y, respectively.

2) If $y \in X_2$, then $O_x := X \setminus \{y\}$ and $O_y := \{y\}$ are disjoint clopen neighborhoods of the points x, y, respectively.

3) If $x, y \in X_0$ or $x, y \in X_1$, then $x \neq y$ impies $x{\restriction}[1, \alpha) \neq y{\restriction}[1, \alpha)$. Consequently, $V_\alpha(x)$ and $V_\alpha(y)$ are disjoint clopen neighborhoods of the points x, y, respectively.

4) If the doubleton $\{x, y\}$ intersects both sets X_0 and X_1, then

$$O_x := V_\alpha(x) \qquad \text{and} \qquad O_y := X \setminus V_\alpha(x)$$

are disjoint clopen neighborhoods of the points x, y, respectively. □

Claim 11. For any cardinal $\kappa < \mathrm{cf}(\lambda)$, the space (X, τ) is a P_κ-space.

Proof. Given any family $\mathcal{U} \subset \tau$ of cardinality

$$|\mathcal{U}| \leqslant \kappa < \mathrm{cf}(\lambda),$$

we should prove that the intersection $\bigcap \mathcal{U}$ belongs to the topology τ. Fix any point $x \in \bigcap \mathcal{U}$. By the definition of the topology τ, for every set $U \in \mathcal{U} \subset \tau$ there exists an ordinal $\alpha_U \in [\|x\|, \lambda)$ such that $V_{\alpha_U}(x) \subseteq U$. Since $|\mathcal{U}| \leqslant \kappa < \mathrm{cf}(\lambda)$, the ordinal $\alpha = \sup\{\alpha_U : U \in \mathcal{U}\}$ is strictly smaller than λ. Since $V_\alpha(x) \subset \bigcap \mathcal{U}$, the set $\bigcap \mathcal{U}$ belongs to the topology τ by the definition of τ. □

Claim 12. If $\lambda = \omega$, then the countable space (X, τ) is metrizable.

Proof. Being Hausdorff and zero-dimensional, the space (X, τ) is regular. If $\lambda = \omega$, then by Claim 8, the space (X, τ) is second-countable. By the Urysohn Metrization Theorem 4.2.9 [3], the space (X, τ) is metrizable. □

Claim 13. (X, τ) is a Lawson semitopological semilattice.

Proof. Given any element $a \in X$, we first prove the continuity of the shift $s_a : X \to X$, $s_a : x \mapsto ax$, at any point $x \in X$. Given any neighborhood $O_{ax} \in \tau$ of ax, we need to find a neighborhood $O_x \in \tau$ of x such that $aO_x \subseteq O_{ax}$. Using Claim 7, find an ordinal $\alpha \in \lambda$ such that $\alpha \geqslant \|ax\| = \max\{\|a\|, \|x\|\}$ and $V_\alpha(ax) \subseteq O_{ax}$. It remains to prove that $aV_\alpha(x) \subseteq V_\alpha(ax)$.

Four cases are possible.

1) $x \in X_2$. In this case $aV_\alpha(x) = a\{x\} = \{ax\} \subseteq V_\alpha(ax)$.

2) $a \in X_0$ or $x \in X_0$. In this case $ax \in X_0$ and for any $z \in V_\alpha(x)$ we have $z{\restriction}[1,\alpha) = x{\restriction}[1,\alpha)$, which implies $az{\restriction}[1,\alpha) = ax{\restriction}[1,\alpha)$ and finally $az \in V_\alpha(ax) \cap X_0$. Therefore, $aV_\alpha(x) \subseteq V_\alpha(ax)$.

3) $a \in X_1$ and $x \in X_1$. In this case $ax \in X_1$ and for any $z \in V_\alpha(x)$ we have

$$z{\restriction}[1,\alpha) = x{\restriction}[1,\alpha) \qquad \text{and} \qquad z([\alpha,\lambda)) \subseteq \{0,2\}.$$

Taking into account that $\|a\| \leqslant \alpha$, we conclude that

$$az{\restriction}[1,\alpha) = ax{\restriction}[1,\alpha) \qquad \text{and} \qquad (az)([\alpha,\lambda)) = z([\alpha,\lambda)) \subseteq \{0,2\},$$

which implies $az \in V_\alpha(ax) \cap X_1$ and $aV_\alpha(x) \subseteq V_\alpha(ax)$.

4) $a \in X_2$ and $x \in X_1$. In this case $ax \in X_1$ and $V_\alpha(x) \subseteq X_1 \cup X_2$. Observe that for any $z \in V_\alpha(x)$ we have

$$z{\restriction}[1,\alpha) = x{\restriction}[1,\alpha) \qquad \text{and} \qquad z([\alpha,\lambda)) \subseteq \{0,2\}.$$

Taking into account that $\|a\| \leqslant \alpha$, we conclude that

$$az{\restriction}[1,\alpha) = ax{\restriction}[1,\alpha) \qquad \text{and} \qquad (az)([\alpha,\lambda)) = z([\alpha,\lambda)) \subseteq \{0,2\}.$$

If $z \in X_1$, then $az \in X_1$ and $az \in V_\alpha(ax) \cap X_1$. If $z \in X_2$, then $\|z\| > \alpha$ and $\|az\| \geqslant \|z\| > \alpha$ witnessing that $az \in V_\alpha(az) \cap X_2$. In both cases we get $az \in V_\alpha(ax)$ and hence $aV_\alpha(x) \subseteq V_\alpha(ax)$.

Therefore, (X,τ) is semitopological semilattice. Since the base

$$\mathcal{B} = \{V_\alpha(x) : x \in X,\ \alpha \in [\|x\|,\lambda)\}$$

of the topology τ consists of open subsemilattices of X, the semitopological semilattice (X,τ) is Lawson. □

Claim 14. The partial order $\leqslant_X := \{(x,y) \in X \times X : xy = x\}$ of X is not closed in $X \times X$.

Proof. For every $\alpha \in \lambda$ consider the elements

$$\mathbf{0}_\alpha : \lambda \to \{0,2\} \quad \text{and} \quad \mathbf{1}_\alpha : \lambda \to \{1,2\}$$

of X, uniquely determined by the conditions $\mathbf{0}_\alpha^{-1}(0) = \{\alpha\} = \mathbf{1}_\alpha^{-1}(1)$. It is clear that $\mathbf{0}_\alpha \leqslant \mathbf{1}_\alpha$ and hence $(\mathbf{1}_0, \mathbf{0}_0) \notin \leqslant_X$. On the other hand, for any ordinal $\alpha \in [1,\lambda)$ we have

$$\{(\mathbf{0}_\beta, \mathbf{1}_\beta) : \beta \in [\alpha,\lambda)\} \subseteq \big(V_\alpha(\mathbf{1}_0) \times V_\alpha(\mathbf{0}_0)\big) \cap \big(\leqslant_X\big),$$

which means that the pair $(\mathbf{1}_0, \mathbf{0}_0) \notin \leqslant_X$ belongs to the closure of $\leqslant_X$. Consequently, the partial order $\leqslant_X$ of X is not closed in $X \times X$. □

Claims 5–14 imply that (X,τ) is a Lawson semitopological semilattice possessing the properties (1)–(6) of Example 4. □

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