



Olympic links in a Chebotarev link

Jun Ueki

Abstract. The Chebotarev law for an infinite link is an equidistribution property about how its components are linked in a group theoretic sense. We overview several properties of a Chebotarev link following the author’s article “Chebotarev links are stably generic”. In addition, we exhibit the density of modulo 2 Olympic links in a Chebotarev link.

Анотація. Закон Чеботарева для нескінченних завузлень це властивість рівномірності розподілу того, як кола завузлені з точки зору теорії груп. Ми розглядаємо кілька властивостей завузлення Чеботарева, слідуючи статті автора “Завузлення Чеботарева є стабільно типовими”. Крім того, ми показуємо щільність по модулю 2 «олімпійського завузлення» в завузленнях Чеботарева.

1. INTRODUCTION

The analogy between knots and prime numbers was initially pointed out by B. Mazur [13] and developed in a systematic manner by M. Kapranov [9], A. Reznikov [23, 24], M. Morishita [18, 19] and others. The Chebotarev law for an infinite link is an equidistribution property about how its components are linked in a group theoretic sense. An infinite link in S^3 obeying the Chebotarev law might be a good analogue of the set of all prime numbers in several senses. In this article, we overview several properties of Chebotarev links mainly following the author’s article [32] and also exhibit the density of modulo 2 Olympic links in a Chebotarev link as a new example of Chebotarev phenomenon.

We first overview the relationship between two analogues in a 3-manifold of the set of rational prime numbers. We recall the definitions and state that if $(K_i)_{i \in \mathbb{N}_{>0}}$ is a sequence of knots obeying the Chebotarev law in the sense of B. Mazur and C. T. McMullen, then $\mathcal{K} = \cup_i K_i$ is a stably generic

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link in the sense of T. Mihara. We outline the proof and also remark our perspective on Idelic class field theory on a 3-manifold and Artin L -functions.

Next, we recall McMullen's examples and discuss the planetary links obtained from fibered hyperbolic links in S^3 . In addition, we recall an analogue of Artin's argument and exhibit the decomposition table in an analogue of a quintic field.

Finally, we recall the notions of (modulo 2) Olympic primes and links and calculate the density of modulo 2 Olympic links in a Chebotarev link.

2. CHEBOTAREV LINKS ARE STABLY GENERIC

Definitions and Theorems. We assume that any 3-manifold is the complement of a finite link in an oriented connected closed 3-manifold. A *knot* K in a 3-manifold M means a tame embedding $S^1 = \mathbb{R}/\mathbb{Z} \hookrightarrow M$ or its image with a natural orientation. A *link* is a countable (finite or infinite) set of disjoint knots. For any manifold X , we denote the interior of X by $\text{Int}X$.

McMullen [15] established a version of the Chebotarev density theorem in which number fields are replaced by 3-manifolds, answering to Mazur's question on the existence of Chebotarev arrangement of knots in [14]. Their definition is described as follows.

Definition 2.1 (Chebotarev law). Let $(K_i) = (K_i)_{i \in \mathbb{N}_{>0}}$ be a sequence of disjoint knots in a 3-manifold M . For each $n \in \mathbb{N}_{>0}$ and $j > n$, we put $L_n = \cup_{i \leq n} K_i$ and denote the conjugacy class of K_j in $\pi_1(M - L_n)$ by $[K_j]$. We say that (K_i) obeys the Chebotarev law if

$$\lim_{\nu \rightarrow \infty} \frac{\#\{n < j \leq \nu \mid \rho([K_j]) = C\}}{\nu} = \frac{\#C}{\#G}$$

holds for any $n \in \mathbb{N}_{>0}$, any surjective homomorphism $\rho : \pi_1(M - L_n) \rightarrow G$ to any finite group, and any conjugacy class $C \subset G$. (The left hand side is the *natural density* of K_i 's with $\rho([K_j]) = C$.)

An infinite link \mathcal{K} is said to be *Chebotarev* if it obeys the Chebotarev law with respect to some order.

On the other hand, Mihara [16] formulated an analogue of idelic class field theory for 3-manifolds by introducing certain infinite links called stably generic links, refining the notion of very admissible links given by Niibo and the author [20, 21], and gave a cohomological interpretation of our previous formulation. Here we describe the definition of a stably generic link, only using ordinary terminology of low dimensional topology:

Definition 2.2 (stably generic link). Let M be a 3-manifold and $\mathcal{K} \neq \emptyset$ a link. The link \mathcal{K} is said to be *generic* if for any finite sublink L of \mathcal{K} , the group $H_1(M - L)$ is generated by components of $\mathcal{K} - L$. The link \mathcal{K} is said to be *stably generic* if for any finite sublink L of \mathcal{K} and for any finite branched cover $h : M' \rightarrow M$ branched over L , the preimage $h^{-1}(\mathcal{K})$ is again a generic link of M' .

The following theorem is due to the author [32, Theorem 3]:

Theorem 2.3. *Let (K_i) be a sequence of disjoint knots in a 3-manifold M obeying the Chebotarev law. Then the link $\mathcal{K} = \cup_i K_i$ is a stably generic link.*

Remark 2.4. For a sequence of knots ordered by length and obeying the Chebotarev law, we may define analogues of Artin L -functions (cf. [1, 22, 27, 28]). In addition, Mihara's refinement allows us to study analogues of ray class fields. We expect that Theorem 2.3 would play a key role to expand an analogue of idèlic class field theory for 3-manifolds, in a direction of analytic number theory, with ample interesting examples. Another analogue in a more general setting are due to J. Kim (see ver.1 of [10]) and others [11].

Outline of the proof. A careful observation of the behavior of knots in a finite cover which is not necessarily Galois (regular) and the lifting property of a continuous map yield the following lemma.

Lemma 2.5. *Let $h : N \rightarrow M$ be a finite (unbranched) cover of 3-manifolds and $K \subset M$ a knot.*

- (1) *Let K' be a connected component of $h^{-1}(K)$ in N . If the restriction map $h|_{K'} : K' \rightarrow K$ is a bijection, then the conjugacy classes satisfy $h_*([K']) \subset [K]$ in $\pi_1(M)$.*
- (2) *If $k \in [K] \cap h_*(\pi_1(N)) \neq \emptyset$, then there exists some connected component K' of $h^{-1}(K)$ such that $h|_{K'} : K' \rightarrow K$ is a bijection and $k \in h_*([K'])$ holds.*

We have an analogue of the Hilbert ramification theory for Galois branched covers of 3-manifolds, in which we describe the behavior of knots (instead of prime ideals) using the language of fundamental groups [31]. For a non-Galois cover $h : N \rightarrow M$ and a knot K in M , the covering degrees of the restriction maps $h|_{K'_i} : K'_i \rightarrow K$ for components of $h^{-1}(K) = \cup_i K'_i$ does not necessarily coincide with each other.

In order to prove Theorem 2.3, we introduce the notion of a weakly Chebotarev link:

Definition 2.6. Let M be a 3-manifold and $\mathcal{K} = \cup_{i \in \mathbb{N}_{>0}} K_i$ a countable link. The link \mathcal{K} in M is said to be *weakly Chebotarev* if for any surjective homomorphism $\rho : \pi_1(M) \rightarrow G$ to any finite group, any conjugacy class C of G is the image $\rho([K_i])$ of the conjugacy class $[K_i] \subset \pi_1(M)$ of some component K_i of \mathcal{K} .

Lemma 2.5 yields the following key lemma.

Lemma 2.7. *If $\mathcal{K} = \cup_i K_i$ in M is weakly Chebotarev, then for any finite (unbranched) cover $h : N \rightarrow M$, the preimage $\mathcal{K}' = \cup_j K'_j$ of \mathcal{K} is again weakly Chebotarev.*

We also have the following lemma.

Lemma 2.8. *If a link $\mathcal{K} = \cup_i K_i$ in a 3-manifold M is weakly Chebotarev, then $H_1(M)$ is generated by components of \mathcal{K} .*

Now Lemmas 2.7, 2.8, and a careful reading of the definition of a stably generic link yield Theorem 2.3.

3. EXAMPLES OF CHEBOTAREV LINKS

McMullen's results. McMullen proved that sequences of knots

$$(K_i) = (K_i)_{i \in \mathbb{N}_{>0}}$$

given in Examples 3.1 below obey the Chebotarev law [15, Theorems 1.1, 1.2]. Hence the union $\mathcal{K} = \cup_i K_i$ of such (K_i) is a stable generic link by Theorem 2.3.

Examples 3.1. (1) Let X be a closed surface of constant negative curvature, let $M = T_1(X)$ denote the unit tangent bundle, and let (K_i) denote the closed orbits of the geodesic flow in M , ordered by length.

(2) Let (K_i) be the closed orbits of any topologically mixing pseudo-Anosov flow on a closed 3-manifold M , ordered by length in a generic metric. We consult [4, 5] for terminology and basic facts related to pseudo-Anosov flows.

In [15], in order to connect symbolic dynamics to finite branched covers, McMullen proved an important lemma that assures that every conjugacy class of G is presented by a closed orbit, and invoked the notion of a Markov section. Then he applied a Chebotarev law for dynamical setting, which was proved by Parry–Pollicott [22, Theorem 8.5] with use of a method of Artin L -functions. We note that special cases of (1) and (2) were initially proved to obey the Chebotarev law by Adachi and Sunada in [27, Proposition II-2-12] and [1, Proposition C], the latter being related to topological entropy.

Planetary links. An interesting example contained in (2) above is *the planetary link* \mathcal{K} of a fibered hyperbolic finite link L in S^3 introduced by Birman and Williams [3], that is, the periodic orbits of the suspension flow of the monodromy map of L . By virtue of McMullen's theorem [15, Theorem 1.2] together with the Nilsen-Thurston uniformization theorem [30, Theorem 0.1], the union $\mathcal{K} \cup L$ obeys the Chebotarev law, if ordered by length. We may prove in a similar way to Miller [17, Proposition 4.2] that for any finite sublink $L' \subset \mathcal{K}$, the union $L \cup L'$ is again hyperbolic. Moreover, we may prove that such $\mathcal{K} \cup L$ is *stably Chebotarev*. See [32, Section 4].

In addition to the Chebotarev law, the planetary link sometimes has another very noteworthy property. Ghrist and others proved that if a link L belongs to a certain large class of links containing the figure-eight knot, then the planetary link of L contains every links [6–8]. Hence we have a sequence (K_i) of knots containing every isotopy class of links and obeying the Chebotarev law. Moreover, as formulated by Kopei in [12], this example satisfies an analogue of the product formula $|a|_\infty \prod_p |a|_p = 1$ ($a \in \overline{\mathbb{Q}}$), where p runs through all the prime numbers and $|a|_p$ denote the p -adic norm with $|p|_p = p^{-1}$. Therefore, the planetary link would give a fundamental setting, when we establish an analogue of number theory on 3-manifolds.

4. ARTIN'S ARGUMENT FOR A_5 -EXTENSIONS

Here we discuss an analogue of a quintic field. We first state the coincidence of the decomposition type of a knot and the cycle type of the monodromy permutation, which is an analogue of Artin's argument in [2] (see also [29, Chapter 16.2, Theorem 2]).

Definition 4.1. (1) For each $n \in \mathbb{N}_{>0}$, we denote the n -th symmetric group by S_n and the n -th alternating group by A_n . *The cycle type of $\sigma \in S_n$ is (f_1, \dots, f_r) if it is the product of disjoint cycles of length $f_1 \geq \dots \geq f_r$.*

(2) For a finite cover $h : N \rightarrow M$ and a knot $K \subset M$, *the decomposition type of K in h is (f_1, \dots, f_r) if the inverse image of K consists of r components as*

$$h^{-1}(K) = \bigcup_{1 \leq i \leq r} K_i$$

with $f_i = \deg(h : K_i \rightarrow K)$ for $i = 1, \dots, r$ and $f_1 \geq \dots \geq f_r$.

We may prove the following proposition in a parallel way to the one in number theory.

Proposition 4.2. *Let $\tilde{h} : W \rightarrow M$ be a finite (unbranched) Galois cover with $G = \text{Gal}(\tilde{h})$, and $h : N \rightarrow M$ a subcover of degree n , which is not necessarily Galois. Let $\rho : G \rightarrow S_n$ denote the monodromy permutation*

induced by putting

$$h^{-1}(b_M) = \{b_1, b_2, \dots, b_n\}$$

with $b_1 = b_N$.

Let $K \subset M$ be a knot and let $z \in G$ be an element of the image of the conjugacy class $[K] \subset \pi_1(M)$ of K under the natural homomorphism

$$\pi_1(M) \twoheadrightarrow \pi_1(M) / \tilde{h}_*(\pi_1(W)) \cong G.$$

Then the cycle type of $\rho(z)$ is (f_1, \dots, f_r) if and only if the decomposition type of K in h is (f_1, \dots, f_r) .

Now we observe the density of knots in a Chebotarev link of each decomposition type in a degree 5 subcover of an A_5 (icosahedral)-cover.

Example 4.3. Let $\mathcal{K} = \cup_i K_i$ be the planetary link of the figure-eight knot in S^3 , obeying the Chebotarev law. Let L be a trefoil in \mathcal{K} , and put $M = S^3 - L$. We have a well-known surjective homomorphism

$$\tau : \pi_1(M) \twoheadrightarrow A_5.$$

Let $\tilde{h} : W \rightarrow M$ denote the corresponding A_5 -cover. (Then the Fox completion \overline{W} of W is a Poincaré 3-sphere, cf. [25].) Let $H < \text{Gal}(h) \cong A_5$ be any subgroup of index 5, and let $h : N \rightarrow M$ denote the corresponding subcover of degree 5. The kernel $\text{Ker}(\rho)$ of the monodromy permutation $\rho : A_5 \rightarrow S_5$ coincides with the normalizer of H in A_5 . Since A_5 is a simple group, $\text{Ker}(\rho) = \{\text{id}\}$ holds and ρ is an injection. By Proposition 4.2 together with the Chebotarev law applied for the composite

$$\rho' : \pi_1(M) \xrightarrow{\tau} A_5 \xrightarrow[\rho]{\cong} \text{Im}(\rho),$$

the number of elements of A_5 of each cycle type and the natural density of K_i 's of each decomposition type are given as follows.

(cycle/decomposition) type	(1,1,1,1,1)	(2,2,1)	(3,1,1)	(5)
number of elements of A_5	1	15	20	24
density of knots in \mathcal{K}	1/60	1/4	1/3	2/5

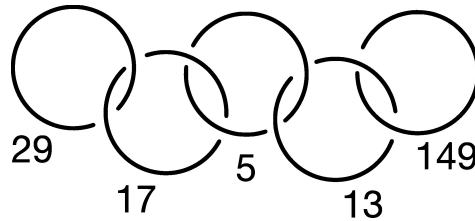
Here, a knot of decomposition type (1,1,1,1,1) is totally decomposed and that of (5) is totally inert. The natural density of K_i 's with property P is defined by

$$\lim_{\nu \rightarrow \infty} \frac{\#\{i \leq \nu \mid K_i \text{ satisfies } P\}}{\nu}.$$

Artin L -functions of symbolic flows due to Parry–Pollicott [22] are regarded as that of Chebotarev links. We may also discuss an analogue of Artin's argument in [2] with use of L -functions associated to the setting in Example 4.3.

5. OLYMPIC PRIMES AND LINKS

The analogy between the Legendre symbol $\left(\frac{p}{q}\right)$ for two prime numbers p, q and the modulo 2 linking number $\text{lk}_2(K, L)$ of a two-component link $K \sqcup L$ is the most classical one in the dictionary of arithmetic topology. Some people say that it was initially noticed by Gauss, and indeed we may find some ideas in his proofs of the quadratic reciprocity $\left(\frac{q^*}{p}\right) = \left(\frac{p}{q}\right)$ for $q^* = (-1)^{\frac{q-1}{2}} q$ which might have come from his study in electromagnetism.



An *Olympic prime* is a 5-tuple $(p_1, p_2, p_3, p_4, p_5)$ of prime numbers satisfying

$$\left(\frac{p_{i+1}^*}{p_i}\right) = -1 \text{ for } i = 1, 2, 3, 4 \quad \text{and} \quad \left(\frac{p_j^*}{p_i}\right) = 1 \text{ for } i + 1 < j.$$

An example is displayed in the figure above. On the first day of Morishita's course in arithmetic topology for undergrads in October 2012, he asked to prove that there are infinitely many Olympic primes. By using Dirichlet's Theorem [26, Chapter VI, Theorem 2], for instance, we may easily calculate that the natural density of Olympic primes is $d = \frac{1}{2^{10}}$, where $10 = \binom{5}{2}$ is "5 choose 2".

Similarly, a *modulo 2 Olympic link* is a link $L_1 \cup \cdots \cup L_5$ with 5 components in S^3 satisfying

$$\text{lk}_2(L_i, L_{i+1}) = 1 \text{ for } i = 1, 2, 3, 4 \quad \text{and} \quad \text{lk}_2(L_i, L_j) = 0 \text{ for } i + 1 < j.$$

Now let $(K_i)_{i \in \mathbb{N}_{>0}}$ be a sequence of disjoint knots in S^3 obeying the Chebotarev law and put $\mathcal{K} = \bigcup_{i \in \mathbb{N}_{>0}} K_i$. For each $n \in \mathbb{N}_{>0}$, let $\bigcup_{1 \leq i \leq n} L_i$ be an n -component sublink of \mathcal{K} and consider the surjective homomorphism

$$\begin{aligned} \rho : \pi_1(S^3 - \bigcup_{1 \leq i \leq n} L_i) &\twoheadrightarrow (\mathbb{Z}/2\mathbb{Z})^n \\ [K_j] &\longmapsto (\text{lk}_2(L_i, K_j))_{i \in \{1, 2, \dots, n\}}. \end{aligned}$$

Then the Chebotarev law yields that the natural density of K_j 's with $\rho([K_j]) = (0, \dots, 0, 1)$ is $d_n = \frac{1}{2^n}$. This fact for $n = 1, 2, 3, 4$ yields that

the natural density of modulo 2 Olympic links in \mathcal{K} is

$$d = \prod_{n \leq 4} d_n = \frac{1}{2^{10}}.$$

The similar argument holds for modulo m linking number for every positive integer m . However, this does not tell whether there exists an Olympic link without modulo.

It might be also interesting to investigate the density of 3-component links with given Milnor's triple linking number.

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Jun Ueki

DEPARTMENT OF MATHEMATICS, SCHOOL OF SYSTEM DESIGN AND TECHNOLOGY, TOKYO
DENKI UNIVERSITY 5 SENJU ASAHI-CHO, ADACHI-KU, 120-8551, TOKYO, JAPAN

Email: uekijun46@gmail.com