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DEVELOPMENT AND ANALYSIS OF METHODS FOR SOLVING TYPICAL PROBLEMS OF RATIONAL DISTRIBUTION OF LIMITED RESOURCES

РОЗРОБКА ТА АНАЛІЗ МЕТОДІВ ВИРІШЕННЯ ТИПОВИХ ЗАДАЧ РАЦІОНАЛЬНОГО РОЗПОДІЛУ ОБМЕЖЕНИХ РЕСУРСІВ.

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Abstract. *The object of research is the problem of rational distribution of limited resources between subsystems of a complex system that function independently. The relevance of the work is determined by the fundamental features of this problem, which take into account the possible variability of the parameters of the environment in which real systems operate. The work sets out tasks for the rational distribution of resources for multiplicative, additive and parametrically defined objective functions, as well as for cases where the parameters of the objective function are not clearly defined. Methods for solving these tasks have been developed. At the same time, it is shown how the desired result can be obtained analytically or numerically. To solve the problem of rational allocation of a one-dimensional resource with multiplicative and additive functions, the method of Lagrange multipliers is used. Under these conditions, the mathematical model of the problem allows it to be reduced to an easily solvable equation. The paper investigates a significantly more complex problem in which the production function for the elements of the system is represented as regression polynomials. In particular, a special case is considered in detail where the parameterised production function depends linearly on two arguments that determine a two-dimensional resource. It is further shown that even a more complex problem with a multidimensional resource can be reduced to an equation that can be solved numerically. Of greatest theoretical interest is the problem of resource allocation for cases where the parameters of the production function are defined imprecisely with (L-R)-type membership functions. The fundamental scientific novelty of the proposed approach to solving the problem consists in the development and use of a technology for reducing the initial fuzzy problem to a clear one by finding and using the membership function of the fuzzy criterion of the problem. At the same time, the unique adaptive properties of (L-R)-type numbers determine good opportunities for the practical use of the proposed methods.*

Анотація. *Об'єкт дослідження – задача раціонального розподілу обмежених ресурсів між підсистемами складної системи, що функціонують незалежно. Актуальність роботи визначається принциповими особливостями цієї задачі, що враховують можливу мінливість параметрів середовища, в якому функціонують реальні системи. У роботі виконано постановки задач раціонального розподілу ресурсів для мультиплікативної, адитивної та параметрично визначеної цільових функцій, а також для випадку, коли параметри цільової функції визначені нечітко. Розроблено методи вирішення цих задач. При цьому показано, як саме шуканий результат може бути отриманий аналітично або чисельно. Для вирішення задачі раціонального розподілу одновимірного ресурсу з мультиплікативною та адитивною функціями використано метод невизначених множників Лагранжа. У цих умовах математична модель задачі дозволяє звести її до рівняння, яке легко вирішується. У роботі досліджено значно складнішу задачу, в якій виробнича функція для елементів системи представлена у вигляді регресійних поліномів. При цьому детально розглянуто окремий випадок, коли параметризована виробнича функція лінійно*

залежить від двох аргументів, що визначають двовимірний ресурс. Далі показано, що навіть більш складна задача з багатовимірним ресурсом зводиться до рівняння, яке можна вирішити чисельно. Найбільший теоретичний інтерес має задача розподілу ресурсу для випадків, коли параметри виробничої функції визначені нечітко з функціями належності (L-R)-типу. Принципова наукова новизна запропонованого підходу до вирішення задачі полягає в розробці та використанні технології зведення вихідної нечіткої задачі до чіткої шляхом пошуку та використання функції належності нечіткого критерію задачі. При цьому унікальні адаптаційні властивості чисел (L-R)-типу визначають хороші можливості для практичного використання запропонованих методів.
Key words: rational allocation of limited resources, parameterisation of production functions, solution methods.
Ключові слова: раціональний розподіл обмежених ресурсів, параметризація виробничих функцій, методи вирішення.

Introduction. The task of rational allocation of limited resources belongs to the class of nonlinear mathematical programming problems. The structural features of such problems determine their belonging to conditional optimisation problems. This paper considers the main types of resource allocation problems in a multi-element system. We illustrate the main features of constructing rational resource allocation problems and methods for solving them using a simple example. Let's introduce a vector $x=(x_1,x_2,...,x_n)$ – distribution of resources across system elements and set $f_j(x_j)=a_j x_j^{\alpha}\ j=1,2,...,n$, single-parameter production functions of system elements. Let us define the multiplicative production function of the system, which determines the criterion for the efficiency of resource allocation

$$f(x)=\prod_{j=1}^{h} a_j x_j^{\alpha}=\prod_{j=1}^{n}(a_j)\prod_{j=1}^{n} x_j^{\alpha}=\alpha_0\prod_{j=1}^{n} x_j^{\alpha}. \quad (1)$$

Let's introduce a restriction on the total amount of resources consumed

$$\sum_{j=1}^{n} x_j = c. \quad (2)$$

Unknown vector $x=(x_1,x_2,...,x_n)$ we will find it using Lagrange's method of undetermined multipliers. Let us introduce the Lagrange function

$$\varphi(x,\lambda)=a_0\prod_{j=1}^{n} x_j^{\alpha} - \lambda\Big(\sum_{j=1}^{n} x_j - c\Big). \quad (3)$$

Next

$$\frac{\delta\varphi(x,\lambda)}{\delta x_{j0}} = a_0\alpha\Big(\prod_{j\ne j_0} x_j^{\alpha}\Big)x_{j_0}^{\alpha-1} - \lambda = 0 \quad (4)$$

or

$$\frac{a_0\alpha}{x_{j_0}}\prod_{j=1}^{n} x_j^{\alpha} - \lambda = 0.$$

From here

$$x_{j_0} = \frac{a_0\alpha}{\lambda}\prod_{j=1}^{n} x_j^{\alpha},\ j_0 = 1,2,...,n. \quad (5)$$

Unknown value λ we will find it from equation (2). We have

$$\sum_{j=1}^{n} x_j = \frac{1}{\lambda} n a_0\alpha\prod_{j=1}^{n} x_j^{\alpha} = c,$$

$$\frac{1}{\lambda} = \frac{C}{n a_0\alpha\prod_{j=1}^{n} x_j^{\alpha}}. \quad (6)$$

Substituting (6) into (5), we obtain

$$x_j = \frac{c}{n a_0\alpha\prod_{j=1}^{n} x_j^{\alpha}}\cdot\alpha_0\alpha\prod_{j=1}^{n} x_j^{\alpha} = \frac{C}{n}\cdot$$

The triviality of the solution obtained is explained by the multiplicative nature of criterion (1) and the extreme simplicity of the production function. Let us now obtain solutions to this problem by introducing another variant of the production function. Let

$$f(x) = \sum_{j=1}^{n} f_j\left(x_j\right) = \sum_{j=1}^{n} a_j x_j^{\alpha} \quad (7)$$

Again, using Lagrange's method of undetermined multipliers, we obtain

$$\varphi(x,\lambda) = \sum_{j=1}^{n} a_j x_j^{\alpha} - \lambda\left(\sum_{j=1}^{n} x_j - c\right)$$

Next

$$\frac{\delta\varphi(x,\lambda)}{\delta x_j} = a_j \alpha x_j^{\alpha-1} - \lambda = 0,$$

$$x_j^{\alpha-1} = \frac{\lambda}{a_j \alpha},\ j = 1,2,\ldots,n,$$

were

$$x_j = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \cdot \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}.$$

Unknown multiplier $\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}}$ we will find it using constraints (2):

$$\sum_{j=1}^{n} x_j = \sum_{j=1}^{n}\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \times \sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} = c,$$

from here

$$\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{c}{\sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}.$$

Then

$$x_j = \frac{c \cdot \left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}} = \frac{c \cdot a_j^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n} a_j^{\frac{1}{1-\alpha}}}. \quad (8)$$

The variety of practical problems leading to the typical schemes (1)-(8) naturally determines the corresponding set of publications.

Analysis of publications on the issue of rational resource allocation. Work [1] considers the problem of multidimensional resource allocation, modelled as a nonlinear integer programming problem. The computational procedure is implemented using the branch and bound method. The effectiveness of the method depends significantly on the dimension of the problem. The same approach—nonlinear integer programming—is used in [2]. In [3], the problem of rational distribution of water resources in river basins is solved. The computational procedure is implemented using a two-level model. The upper level of the model is used to distribute water resources, while the lower level ensures maximisation of expected benefits. The specific formulation of the computational procedure for the problem being solved makes it difficult to use the proposed method to solve other problems. In [4], a heuristic algorithm is proposed to solve the problem of rational resource allocation, but its accuracy is not discussed. The main content of [5] consists of solving the problem of resource allocation in two stages. In the first stage, it is proposed to obtain a set of feasible solutions, from which the optimal one is selected in the second stage. It is clear that the effectiveness of the proposed technology significantly depends on the feasibility threshold, the correctness of which is a separate task. In [6], a class of resource allocation problems for service systems is considered, in which the objective function and constraints are represented probabilistically. The Bayesian approach is used to take into account a priori data and assess the effectiveness of possible solutions. The proposed approach is strictly tied to the formulation of a specific problem to be solved. The main content of [7] is related to management and recovery issues arising from the consequences of the COVID-19 pandemic. The corresponding mathematical model of resource distribution does not contain any novelty. In [8], a goal space and a solution space are introduced to solve the problem of rational resource distribution. A heuristic procedure for obtaining an initial solution is proposed, which is iteratively improved. Work [9] is devoted to the consideration of the problem of distributing multiple resources with partial preferences, including those specified probabilistically. Examples of specific problems solved by known methods are given. In [10], the problem of multidimensional resource allocation for optimising throughput in the Internet of Things system is solved. A heuristic algorithm is used to solve the resulting nonlinear programming problem.

An analysis of well-known publications on solving problems of rational resource allocation leads to the conclusion that this issue remains relevant.

Purpose of the study. Development and analysis of methods for solving typical problems of rational distribution of limited resources.

Materials and methods for solving the problem. Let us now solve the initial problem using a more informative representation than before for the production function of the system elements and an additive criterion for its efficiency.

Let's introduce

$$f_j|x| = a_j x_j^{\alpha_j}$$

and

$$f(x) = \sum_{j=1}^{n} f_j(x_j) = \sum_{j=1}^{n} a_j x_j^{\alpha_j}. \tag{9}$$

In this case, the Lagrange function has the following form:

$$\varphi(x_1\lambda) = \sum_{j=1}^{n} a_j x_j^{\alpha_j} - \lambda\left(\sum_{j=1}^{n} x_j - c\right).$$

Next

$$\frac{\gamma\varphi(x_1\lambda)}{\delta x_j} = a_j\lambda_j x_j^{\alpha_j-1} - \lambda = 0,\ j = 1,2,...,n.$$

from here

$$x_j^{\alpha_j-1} = \frac{\lambda}{a_j\alpha_j},\ x_j = \frac{\lambda}{a_j\alpha_j}^{\frac{1}{\alpha_j-1}} = (\lambda)^{\frac{1}{\alpha_j-1}}\cdot\left(\frac{1}{\alpha_j-1}\right)^{\frac{1}{\alpha_j-1}}.$$

Let's find the multiplier $(\lambda)^{\frac{1}{\alpha_j-1}}$, using (2).

$$\sum_{j=1}^{n} x_j = \sum_{j=1}^{n}\left(\frac{\lambda}{a_j\alpha_j}\right)^{\frac{1}{\alpha_j-1}} = C. \tag{10}$$

The resulting nonlinear relative to λ The equation is solved numerically. A simple analytical solution can be found in the special case when $\alpha_j = \alpha,\ j = 1,2,..,n.$

Then relation (9) takes the form:

$$x_j = x^{\frac{1}{\alpha-1}}\cdot\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}.$$

At the same time

$$\sum_{j=1}^{n} x_j = (\lambda)^{\frac{1}{\alpha-1}}\cdot\sum_{j=1}^{n}\left(\frac{1}{\alpha_j\alpha}\right)^{\frac{1}{\alpha-1}} = C.$$

Then

$$(\lambda)^{\frac{1}{\alpha-1}} = \frac{C}{\sum_{j=1}^{n}\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}}$$

and

$$x_j = \frac{C\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n}\left(\frac{1}{a_j\alpha}\right)^{\frac{1}{\alpha-1}}},\ j = 1,2,...,n. \tag{11}$$

Let us bring (11) to a form that is more convenient for calculations:

$$x_j = C\frac{(a_j\alpha)^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n}(a_j\alpha)^{\frac{1}{1-\alpha}}} = C\frac{a_j^{\frac{1}{1-\alpha}}}{\sum_{j=1}^{n}a_j^{\frac{1}{1-\alpha}}},$$

which, of course, coincides with (8).

Let us now consider the problem of distributing a two-dimensional resource with single-parameter production functions of system elements. Let us introduce the production function of system elements

$$f_j(x_1x_2) = a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha};\ j = 1,2\ldots n,$$

$$f(x_1x_2) = \sum_{j=1}^{n} f_j(x_1x_2) =$$

$$= \sum_{j=1}^{n}(a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha}). \tag{12}$$

The restriction on the two-dimensional resource is as follows:

$$\sum_{j=1}^{n} x_{1j} = c_1, \tag{13}$$

$$\sum_{j=1}^{n} x_{2j} = c_2. \tag{14}$$

Next

$$\varphi(x_1x_2) = \sum_{j=1}^{n}(a_{1j}x_{1j}^{\alpha} + a_{2j}x_{2j}^{\alpha}) -$$

$$-\lambda_1(\sum_{j=1}^{n}x_{1j} - c_1) - \lambda_2(\sum_{j=1}^{n}x_{2j} - c_2)$$

$$\frac{\gamma\varphi(x_1x_2)}{\delta x_{1j}} = \alpha a_{1j}x_{1j}^{\alpha-1} - \lambda_1 = 0, \quad j = 1,2,\ldots n \tag{15}$$

$$\alpha a_{1j}x_{1j}^{\alpha-1} = \lambda_1, \quad x_{1j} = \left(\frac{\lambda_1}{\alpha a_{1j}}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\lambda}{\alpha}\right)\bullet\left(\frac{1}{a_{1j}}\right). \tag{16}$$

Let's find it λ_1, using (13),

$$\sum_{j=1}^{n} x_{1j} = \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} \bullet \sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}} = c_1, \tag{17}$$

were

$$\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{c_1}{\sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}}, \tag{18}$$

$$x_{1j} = \frac{c_1\left(\frac{1}{a_{1j}}\right)^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n}\left(\frac{1}{a_j}\right)^{\frac{1}{\alpha-1}}} = \frac{c_2 a_{1j}^{\frac{1}{\alpha-1}}}{\sum_{j}^{n}(a_{1j})^{\frac{1}{\alpha-1}}}, \quad j = 1,2,\ldots n. \tag{19}$$

Repeat steps (15) to (18) for x_{2j}, we will obtain a similar result:

$$x_{2j} = \frac{c_2 a_{2j}^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{n}(a_{2j})^{\frac{1}{\alpha-1}}}, \quad j = 1,2,\ldots,n. \tag{20}$$

Let us now solve the problem of optimal allocation of a two-dimensional resource for more informative production functions of system elements.

Let's introduce

$$f_2(x_2x_2) = a_{1j}x_{1j}^{\alpha_{1j}} + a_{2j}x_{2j}^{\alpha_{2j}};$$
$$F(x_1x_2) = \sum_{j=1}^{n} f_j(x_1x_2) = \sum_{j=1}^{n}(a_{1j}x_{1j}^{\alpha_{1j}} + a_{2j}x_{2j}^{\alpha_{2j}}). \quad (21)$$

Let the resource volume restriction still be of the form (13), (14).

Next,

$$\varphi(x_1x_2) = \sum_{j=1}^{n}(a_{1j}x_{1j}^{\alpha_{1j}} + a_{2j}x_{2j}^{\alpha_{2j}}) -$$
$$-\lambda_1\left(\sum_{j=1}^{n}x_{1j} - c_1\right) - \left(\sum_{j=1}^{n}x_{2j} - c_2\right),$$
$$\frac{\delta\varphi\ (x_1x_2)}{\partial x_{1j}} = a_{1j}\alpha_{1j}x_{1j}^{\alpha_{1j}-1} - \lambda_1 = 0, \quad j = 1,2,\dots,n,$$
$$x_{1j}^{\alpha_{1j}-1} = \frac{x_1}{a_{1j}\alpha_{1j}};$$
$$x_{1j} = \lambda_1^{\frac{1}{\alpha_{1j}-1}} \bullet \left(\frac{1}{a_{1j}\alpha_{1j}}\right)^{\frac{1}{\alpha_{1j}-1}}.$$

Now

$$\sum_{j=1}^{n}x_{1j} = \sum_{j=1}^{n}(\lambda_1)^{\frac{1}{\alpha_{1j}-1}} \bullet \left(\frac{1}{a_{1j}\alpha_{1j}}\right)^{\frac{1}{\alpha_{1j}-1}} = c_1. \quad (22)$$

Performing the same actions with the variable x_{2j}, we obtain a similar equation for x_2:

$$\sum_{j=1}^{n}x_{2j} = \sum_{j=1}^{n}(\lambda_2)^{\frac{1}{\alpha_{2j}-1}} \bullet \left(\frac{1}{a_{2j}\alpha_{2j}}\right)^{\frac{1}{\alpha_{2j}-1}} = c_2 \quad (23)$$

Equations (22) and (23) relative to λ_1 and λ_2 are solved numerically, enabling the calculation of optimal values $x_{1j}, x_{2j}, j = 1,2,\dots,n$.

Finally, let us consider the problem of rational resource allocation under conditions of uncertain initial data.

Let us introduce a single-parameter production function j - th element of the system:

$f_j(x_j) = a_j x_j^{\alpha}, j = 1,2,\dots,n.$

Let us now assume that the coefficients a_j there are fuzzy numbers with membership functions (L,R) that is, $a_j =< \mu_j, \alpha_j, \beta_j, j = 1,2,\dots,n.$

Now let's set the task of finding a non-negative set $X = (x_1, x_2, \dots, x_n)$, maximizing the objective function

$$F(X,A) = \sum_{j=1}^{n} a_j x_j^{\alpha}, A = (a_1, a_2, \dots, a_n) \quad (24)$$

and satisfying the constraint

$$\sum_{j=1}^{n} x_j = C. \quad (25)$$

We will obtain a solution to this fuzzy problem by reducing it to an adequate clear mathematical programming problem. In this case, for fuzzy parameters $a_j,\ j = 1,2,..,n$ a certain threshold value is introduced P level of belonging. Now, problems (25) and (26) boil down to the following: find sets $(X,\ A)$, $A = a_1, a_2, \dots, a_n$, maximizing $F(X,A)$, satisfying constraint (26) and additional constraints.

$$\mu(a_j) \geq P,\ j = 1,2,\dots,n \quad (26)$$

The point of this approach is that the resulting solution belongs to the set of alternatives that maximize (25) with a degree of no less than P. The shortcomings of this approach are obvious. First, the resulting problem is significantly more complex than the original one: it contains twice as many variables. Second, it is unclear how to choose the value of P. Similarly, it is unclear how the solution to the problem depends on the chosen value P.

In this regard, let us consider a different approach to solving the problem. In the course of its implementation, using the rules for performing operations on fuzzy numbers, taking into account membership functions $\mu(a_j)$, $j = 1,2,...,n$, the membership function is sought $\mu(f(x_1 a))$ objective function $f(x_1 a)$. Then a specific value is selected. $p < 1$ fuzzy membership level $f(x_1 a)$ and the equation is solved

$$\mu(f(x_1 a)) = \mu(y) = p. \tag{27}$$

This equation has two roots:

$$y_1 = \mu_1^{-1}(p), y_2 = \mu_2^{-1}(p).$$

Next, one of these roots is selected, for example, the first one, and the task of finding a set is formulated $x^{\forall}$, maximising $y_1(x)$ and satisfying constraint (26). The resulting solution x^* corresponds to the membership function, maximally shifted to the region of large values of the objective function. Let us apply this technology to solve problems (25) and (26). Let the membership functions of fuzzy numbers a_j, $j - 1,2,...,n$, – Gaussian and have the form:

$$\mu_j(a_j) = exp\left\{-\frac{(a_j - a_j^{(0)})^2}{2\delta_j^2}\right\}.$$

For any set x The objective function (25) is a fuzzy number with a membership function $\mu(f(x_1 a)) = exp\left\{-\frac{(y - y^{(0)}(x))^2}{2\delta^2(x)}\right\}$,

were

$$y^{(0)}(x) = \sum_{j=1}^{n} a_j^{(0)} x_j^{\alpha}, \delta_y^2(x) = \sum_{j=1}^{n} \delta_j^2 x_j^{2\alpha}.$$

In accordance with the proposed technology, we will set the value of the membership function at the level of p and solve the equation.

$$\mu(y) = exp\left\{-\frac{\left(y - y^{(0)}(x)\right)^2}{2\delta_y^2(x)}\right\} = p. \tag{28}$$

This equation has two roots:

$$y_1 = y^{(0)}(x) - \left(\delta_y^2(x) ln\frac{1}{p^2}\right)^{\frac{1}{2}},$$

$$y_2 = y^{(0)}(x) + \left(\delta_y^2(x) ln\frac{1}{p^2}\right).$$

Choosing the smaller of these roots, we introduce the objective function

$$y^*(x) = \sum_{j=1}^{n} a_j^{(0)} x_j^{\alpha} - s\left(\sum_{j=1}^{n} \delta_j^2 x_j^{2\alpha}\right)^{\frac{1}{2}}, s = \left(ln\frac{1}{p^2}\right)^{\frac{1}{2}} \tag{29}$$

Now the task of finding a set is formulated x^*, maximizing (29) and satisfying the constraint (26), which can be easily solved, for example, using the Nelder-Mead method. The clear set obtained as a result of solving this problem x^* corresponds to a fuzzy value with a membership function

$$\mu(x^*) = exp\left\{-\frac{\left(y - \sum_{j=1}^{n} a_j (x_j^*)^{\alpha}\right)^2}{\alpha\sum_{j=1}^{n} \delta_j^2 (x_j^*)^{2\alpha}}\right\},$$

shifted as far to the right as possible, which was required.

Conclusions

1. Thus, methods for solving the main types of problems involving the distribution of one-dimensional and multidimensional resources have been considered.
2. The direction of further research is the development of methods for solving the resource allocation problem in situations where not only the parameters of the production functions of the system elements, but also the numerical values of the multidimensional resource are specified imprecisely.

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