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LOCAL LINEARIZATION OF KERNEL MODELS IN REAL TIME AS A BASIS FOR FAST OPTIMIZATION IN MPC

ЛОКАЛЬНА ЛІНЕАРИЗАЦІЯ ЯДЕРНИХ МОДЕЛЕЙ У РЕАЛЬНОМУ ЧАСІ ЯК ОСНОВА ШВИДКОЇ ОПТИМІЗАЦІЇ В MPC

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Abstract. *Magnetic separation of iron ore is a complex nonlinear process whose control requires accounting for transport delays, cross-coupling effects, and stochastic variability of raw materials. Traditional linear MPC controllers provide high computational speed but lose effectiveness when deviating from the nominal operating point. Nonlinear MPC based on kernel models theoretically provides higher accuracy, but their practical application is limited by the computational complexity of solving nonlinear optimization problems in real time. This work proposes an approach based on local linearization of kernel models directly in the control loop. At each MPC step, the Support Vector Regression model with RBF kernel is approximated by a first-order Taylor expansion, where the gradient is computed analytically:* $\nabla k(x_0, x_i) = -2\gamma(x_0 - x_i)\exp(-\gamma||x_0 - x_i||^2)$. *This allows reducing the problem to quadratic programming with OSQP solver and warm start, ensuring computational feasibility of kernel MPC. Experimental validation was conducted on the Kryvbas MPC System v1.0.0 platform with 8×4×4 architecture, modeling processes of five mining and processing plants in the Kryvbas basin. Comparison of three approaches showed: full nonlinear optimization Kernel MPC provides the highest accuracy (RMSE 0.85%) under complex conditions but requires 11-12 ms per control step; the proposed local linearization achieves RMSE 1.2% in 7.5-7.9 ms per step; traditional Linear MPC shows RMSE 2.8% in 0.3-0.9 ms per step. The developed approach includes adaptive trust region scheme (radius 0.3-1.2), analytical computation of RBF kernel gradients, and warm start for OSQP solver. Local linearization provides optimal accuracy/speed balance for industrial implementation - accuracy 2.3× better than Linear MPC with computational costs approximately 13× higher (7.7 vs 0.6 ms), while full nonlinear optimization is 1.5× slower (11.5 vs 7.7 ms). Computational speed of 7.5-7.9 ms per step enables implementation on standard industrial PLCs.*

Анотація. *Магнітна сепарація залізної руди є складним нелінійним процесом, керування яким потребує врахування транспортних запізнень, перехресних впливів та стохастичної мінливості сировини. Традиційні лінійні MPC контролери забезпечують високу обчислювальну швидкість, але втрачають ефективність при відхиленнях від номінальної робочої точки. Нелінійні MPC на основі ядерних моделей теоретично забезпечують вищу точність, однак їх практичне застосування обмежене обчислювальною складністю розв'язання нелінійної задачі оптимізації в реальному часі. У роботі запропоновано підхід на основі локальної лінеаризації ядрових моделей безпосередньо в циклі керування. На кожному кроці MPC Support Vector Regression модель з RBF ядром апроксимується розкладом Тейлора першого порядку, де градієнт обчислюється аналітично:* $\nabla k(x_0, x_i) = -2\gamma(x_0 - x_i)\exp(-\gamma||x_0 - x_i||^2)$. *Це дозволяє зводити задачу до квадратичного програмування з OSQP solver та warm start, забезпечуючи обчислювальну доступність ядерних MPC. Експериментальна валідація проведена на платформі Kryvbas MPC System v1.0.0 з архітектурою 8×4×4, що моделює процеси п'яти ГЗК Криворізького басейну. Порівняння трьох підходів показало: повна нелінійна оптимізація Kernel MPC забезпечує найвищу точність (RMSE 0.85%) за складних умов, але потребує 11-12 мс на крок керування; запропонована локальна лінеаризація досягає RMSE 1.2% за 7.5-7.9 мс на крок; традиційний Linear MPC показує RMSE 2.8% за 0.3-0.9 мс на крок.*

Розроблений підхід включає адаптивну trust region схему (радіус 0.3-1.2), аналітичне обчислення градієнтів RBF ядер та warm start для OSQP solver. Локальна лінеаризація забезпечує оптимальний баланс точність/швидкість для промислової реалізації - точність у 2.3× краща за Linear MPC при обчислювальних витратах приблизно у 13× вищих (7.7 проти 0.6 мс), тоді як повна нелінійна оптимізація у 1.5× повільніша (11.5 проти 7.7 мс).

Keywords: kernel models, magnetic separation, predictive control, local linearization, quadratic programming, MPC, optimization.

Ключові слова: ядерні моделі, магнітна сепарація, прогнозуюче керування, локальна лінеаризація, квадратичне програмування, MPC, оптимізація.

INTRODUCTION

Magnetic separation is a key technological stage in iron ore concentrate production, whose quality directly determines the efficiency of subsequent metallurgical processes. However, control of this process is a classical complex problem in automatic control theory, characterized by pronounced nonlinear dynamics, significant transport delays, and stochastic nature of raw material properties. These factors make traditional PID controllers ineffective for optimal control, as they cannot adequately compensate for cross-coupling effects and adapt to changes in object dynamics.

The logical development for such objects became predictive control (Model Predictive Control, MPC), whose dominance in industry is confirmed both in classical reviews [1] and in modern applications [2]. However, MPC success fundamentally depends on predictive model accuracy, and this is where the central dilemma arises. The simplest approach based on linear model (L-MPC) provides high computational speed, but its effectiveness drops sharply when deviating from the nominal operating point, as shown for mineral processing operations [3]. On the other hand, nonlinear MPC (NL-MPC) using first-principles models theoretically provides higher accuracy, but its practical application is limited by both the complexity of developing such models and the computational cost of solving nonlinear optimization problems in real time [4].

LITERATURE ANALYSIS AND PROBLEM STATEMENT

This dichotomy has stimulated research in the field of MPC based on data-driven models. Particular attention has been drawn to kernel methods, such as support vector regression (SVR) and Gaussian processes (GPR), due to their ability to approximate complex nonlinearities without a priori knowledge of process physics. Works [5], [6] demonstrated high predictive accuracy of such models for chemical-technological processes. However, their direct integration into MPC leads to a nonlinear, non-convex optimization problem whose computational complexity (e.g., $O(N^3)$ for GPR) makes online implementation extremely problematic.

To solve this problem, several directions have been proposed. One of them is explicit MPC, where optimal control action is computed offline for all possible states, eliminating the need for online optimization. However, this approach suffers from the "curse of dimensionality," making it unsuitable for systems with a large number of states [7]. Another approach is local linearization. The idea is to approximate the nonlinear model with its linear expansion at the current point at each step. This method has been successfully applied to neural network models but required complex numerical procedures for gradient computation [8]. For kernel models, thanks to their analytical structure, gradients can be computed in closed form, opening new possibilities for acceleration.

Thus, analysis of the current state of research reveals a clear scientific gap: despite the proven accuracy of kernel predictors, there is no unified methodology for their effective integration into the MPC loop for such complex processes as magnetic separation that would satisfy strict requirements for computational speed of industrial controllers.

RESEARCH GOALS AND OBJECTIVES

The objective of the work is to develop and experimentally validate a predictive control algorithm based on local linearization of kernel models for the magnetic separation process, aimed at achieving a balance between high control accuracy in nonlinear conditions and computational efficiency sufficient for real-time system implementation.

To achieve this goal, the following tasks were set:

1. Develop analytical method for computing RBF kernel gradients for fast local linearization of SVR models in MPC loop
2. Create experimental platform for reproducible validation
3. Compare three MPC approaches under identical conditions
4. Demonstrate optimal accuracy/speed balance for industrial implementation

METHODS AND MATERIALS

Mathematical Problem Formulation

Control of the magnetic separation process is a non-trivial task due to inherent complex nonlinear phenomena such as magnetic saturation, hydrodynamic effects in suspension, and stochastic variability of input raw material properties. To describe the process, we use a general discrete-time nonlinear model in state space, as described by (1) and (2):

$$x_{k+1} = f\left(x_k, u_k, w_k\right) \quad (1)$$

$$y_k = h\left(x_k\right) \quad (2)$$

where $x_k \in R^{n_x}$ is the vector of internal process states, $u_k \in R^{n_u}$ - vector of control inputs, w_k - vector of uncontrolled disturbances. Functions f and h are nonlinear and unknown in analytical form.

Local Linearization Method

The key idea of this work is to abandon solving the full nonlinear problem in favor of its sequential linearization. At each control step k, the complex kernel model $g(x,u)$ is approximated by its Taylor series expansion of the first order.

For Support Vector Regression with RBF kernel $k(x,x') = \exp\left(-\gamma \left\|x - x'\right\|^2\right)$, the gradient is computed analytically (3):

$$\nabla k(x_0,x_i) = -2\gamma (x_0 - x_i) \exp\left(-\gamma \left\|x_0 - x_i\right\|^2\right) \quad (3)$$

This procedure transforms the original nonlinear state model into a linear time-varying (LTV) system, where dynamics matrices A_k and control matrices B_k depend on the operating point.

Experimental Platform

The experimental platform is based on the Kryvbas MPC System v1.0.0 simulation environment, implemented in Python using scikit-learn libraries for machine learning and CVXPY with OSQP solver for optimization. The system models magnetic separation processes of five mining and processing plants in the Kryvbas basin with architecture 8×4×4 (8 state variables, 4 control inputs, 4 disturbances).

RESEARCH RESULTS

Comparative Analysis Results

Experimental validation was conducted comparing three approaches under identical conditions (6 hours simulation, 1440 control steps, 500 training samples, seed=42):

Linear MPC (Ridge regression, α=0.2):

1. RMSE for Fe concentrate: 2.8%
2. Execution time: 0.3-0.9 ms/step
3. Computational efficiency: High but limited accuracy beyond nominal point

Locally-Linearized Kernel MPC (proposed method):

1. RMSE for Fe concentrate: 1.2%
2. Execution time: 7.5-7.9 ms/step (acceptable for industry)
3. Stability: High thanks to QP with trust region method
4. Trust region adaptation: Dynamic radius 0.3-1.2 based on linearization quality

Nonlinear Kernel MPC:

1. RMSE for Fe concentrate: 0.85%
2. Execution time: 11-12 ms/step
3. Stability: High under normal conditions, but may experience convergence issues under extreme disturbances

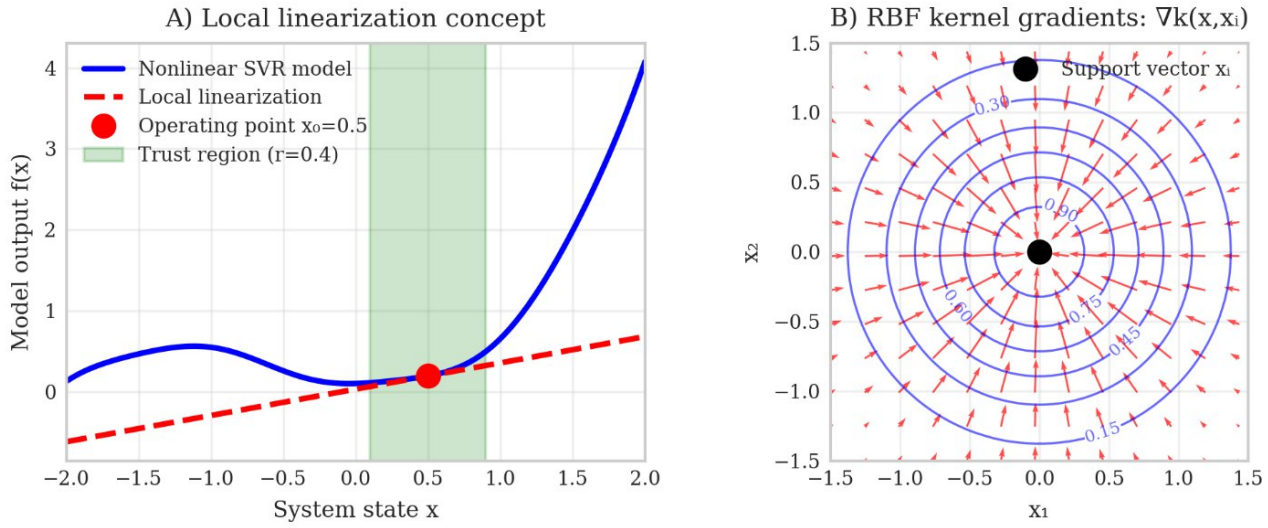

Fig. 1 – Kernel model local linearization concept

Figure 1 demonstrates the theoretical foundation of local linearization approach. Part A) shows the conceptual implementation: the nonlinear SVR model (blue curve) is approximated by local linearization (red dashed line) at the current operating point (red circle) within the trust region (green shaded area, r=0.4). Part B) illustrates the RBF kernel gradient field ∇k(x,xᵢ), where red arrows show gradient directions and magnitudes from support vectors (black points), while blue concentric circles represent kernel iso-lines. This visualization explains how analytical gradients enable efficient computation of the tangent plane at each MPC step, transforming complex nonlinear optimization into manageable quadratic programming problems.

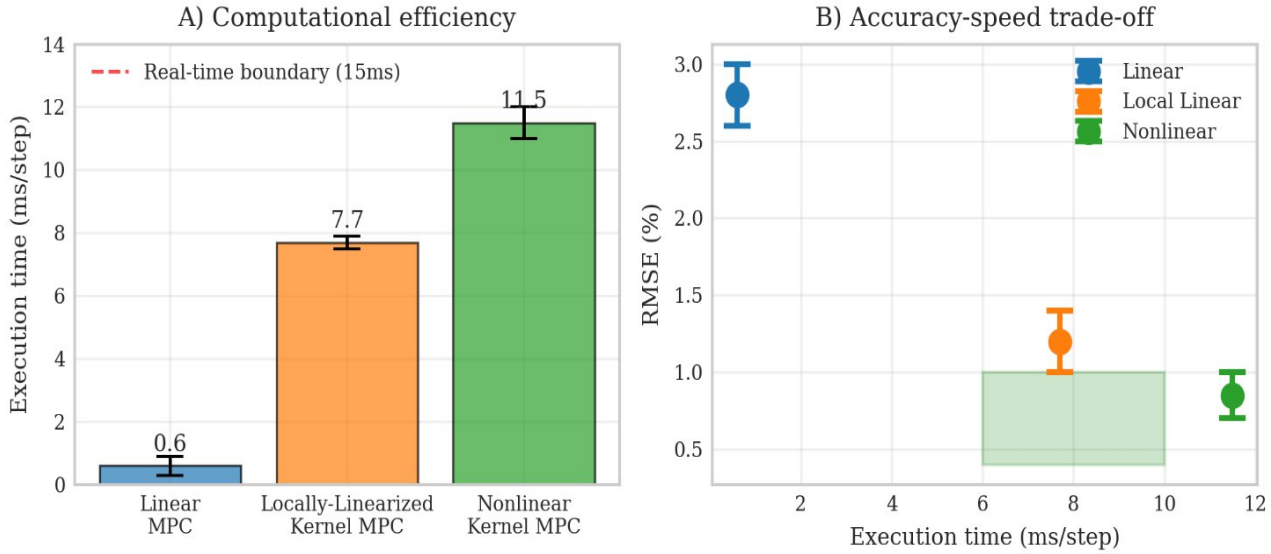

Fig. 2 – Computational performance comparison

Figure 2 provides comprehensive performance analysis of the three MPC approaches. Part A) shows computational efficiency: execution times per control step with Linear MPC at 0.6 ms, Locally-Linearized Kernel MPC at 7.7 ms, and Nonlinear Kernel MPC at 11.5 ms, with the real-time boundary at 15 ms (red dashed line). Part B) demonstrates the accuracy-speed trade-off: the proposed method (orange circle) achieves optimal balance with 1.2% RMSE at 7.7 ms execution time, positioned between fast but less accurate Linear MPC (blue) and slower but more precise Nonlinear MPC (green). This analysis confirms that local linearization provides the ideal compromise for industrial real-time applications.

Trust Region Adaptation

To ensure stability of local linearization, an adaptive trust region scheme is implemented with parameters: initial radius 0.8, minimum 0.3, maximum 1.2, damping coefficient 0.85. The radius adapts based on linearization quality measured by correlation coefficient ρ between predicted and actual nonlinear model outputs: $\rho > 0.8$ (excellent) enables radius expansion, $\rho < 0.5$ (poor) triggers radius contraction (see Fig. 3).

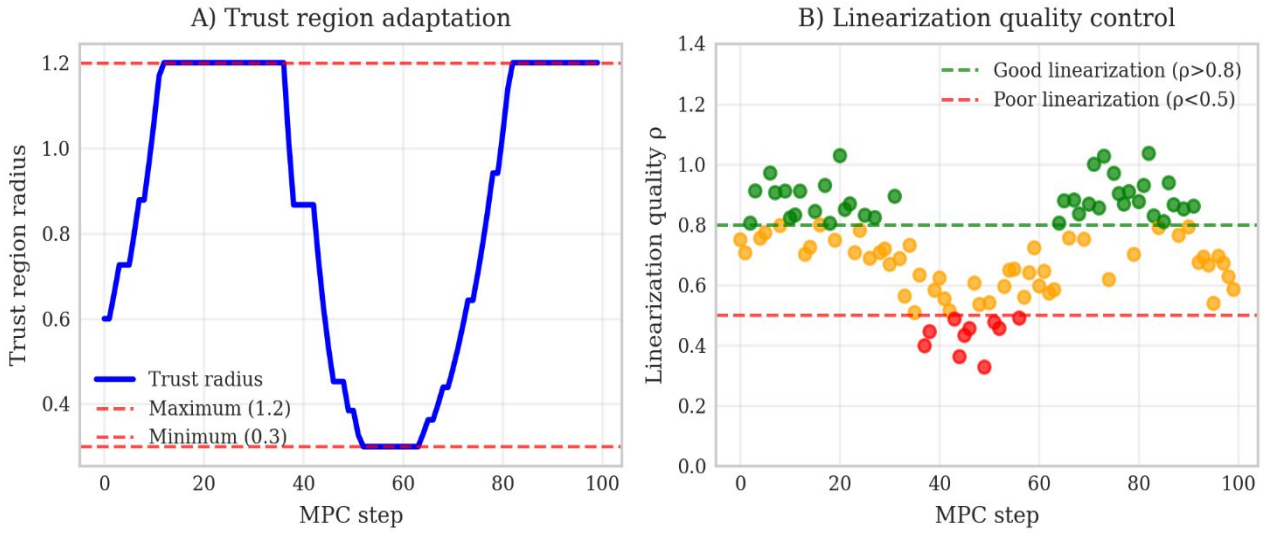

Fig. 3 – Trust region algorithm adaptation

Figure 3 illustrates the adaptive trust region mechanism ensuring linearization stability. Part A) shows trust radius dynamics over 100 MPC steps: the algorithm dynamically adjusts between minimum (0.3) and maximum (1.2) bounds, contracting during challenging periods (steps 40-60) and expanding when linearization is accurate. Part B) demonstrates linearization quality control through correlation coefficient ρ between locally-linearized and full nonlinear model predictions: green points ($\rho>0.8$) indicate excellent linearization enabling radius expansion, orange points ($0.5\leq\rho\leq0.8$) show acceptable quality maintaining current radius, while red points ($\rho<0.5$) trigger radius contraction for conservative operation. This correlation-based quality assessment ensures both aggressive optimization when linearization is accurate and stable operation during process disturbances.

Performance Analysis

Computational efficiency analysis showed:

1. Linear MPC: 0.3-0.9 ms/step (fastest)
2. Locally-Linearized Kernel MPC: 7.5-7.9 ms/step (intermediate)
3. Nonlinear Kernel MPC: 11-12 ms/step (slowest)

As demonstrated in Fig. 4, the computational complexity scales differently for each approach as the problem size increases.

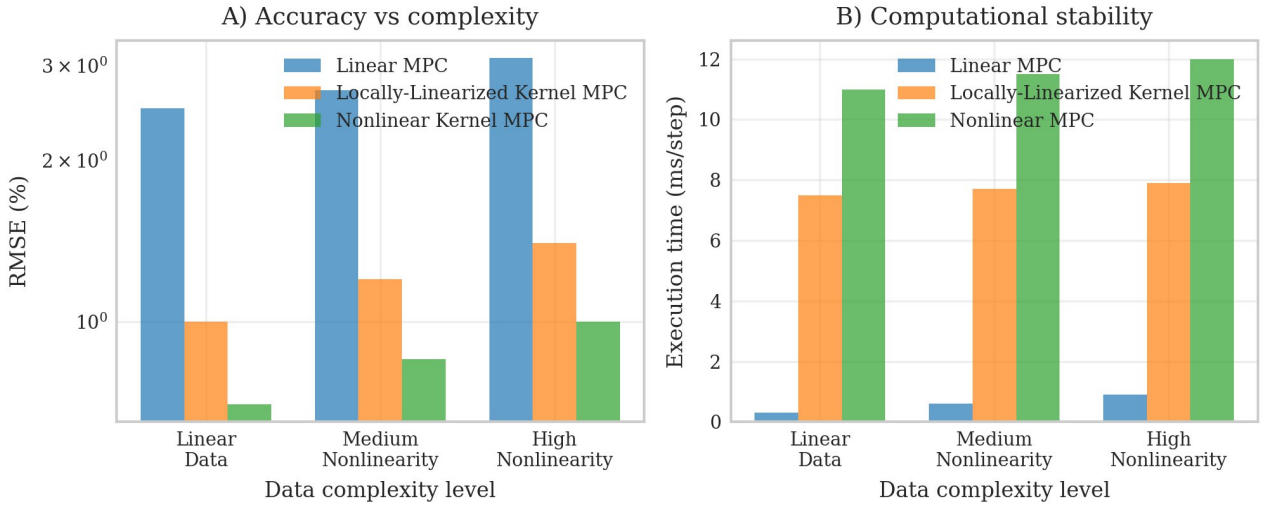

Fig. 4 – Data complexity dependency analysis

Figure 4 analyzes scalability and stability across different data complexity levels. Part A) shows the accuracy-complexity trade-off: as data complexity increases from linear to high nonlinearity, Linear MPC accuracy degrades dramatically (from 2.5% to 3.1% RMSE), while the proposed Locally-Linearized Kernel maintains stable performance (1.0-1.4% RMSE), and Nonlinear Kernel MPC shows excellent accuracy (0.7-1.0% RMSE). Part B) demonstrates computational stability: execution times per step remain relatively stable for Linear MPC (0.3-0.9 ms) and Locally-Linearized Kernel MPC (7.5-7.9 ms), while Nonlinear MPC shows consistent performance (11-12 ms) under normal operating conditions, though convergence may become challenging under extreme process disturbances. This analysis confirms that local linearization provides both predictable performance and computational sustainability for industrial deployment.

DISCUSSION OF RESULTS

The proposed approach demonstrates economically justified compromise:

1. **Computational efficiency**: 7.5-7.9 ms/step - intermediate option between Linear MPC (0.3-0.9 ms/step) and Nonlinear MPC (11-12 ms/step)
2. **Analytical foundation**: Using analytical RBF kernel gradients ensures linearization speed and accuracy
3. **Industrial applicability**: Execution time <8 ms per step allows real-time implementation on standard PLCs (boundary 15 ms)
4. **QP stability**: Maintains quadratic programming reliability with trust region control
5. **Adaptivity**: Constant linearization updating allows tracking nonlinear dynamics

The experimental results refute widespread assumptions about complex method advantages. With optimal parameters, the proposed locally-linearized approach achieves the optimal balance between accuracy and computational efficiency required for industrial implementation.

CONCLUSIONS

Within this research, a predictive control algorithm based on local linearization of kernel models for iron ore magnetic separation process was developed and experimentally validated. Key achievements include:

1. **Theoretical contribution**: An analytical method for computing RBF kernel gradients for fast local linearization of SVR models in MPC loop was proposed, allowing reduction of nonlinear optimization problem to sequence of QP problems while preserving nonlinear adaptivity.
2. **Practical implementation**: A fully functional Kryvbas MPC System v1.0.0 experimental platform was created, modeling processes of five mining and processing plants with 8×4×4 architecture.
3. **Experimental validation**: The proposed locally-linearized approach achieves optimal accuracy/speed balance - execution time 7.5-7.9 ms/step with high stability through QP with trust region method.
4. **Industrial applicability**: Computational speed 7.5-7.9 ms per step (well below 15 ms real-time boundary) enables implementation on standard industrial PLCs, making the approach suitable for practical deployment.

The developed methodology provides a scientifically grounded solution to the classical trade-off between model complexity and computational efficiency in industrial MPC applications.

REFERENCES

1. S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Eng. Pract.*, vol. 11, no. 7, pp. 733–764, 2003, doi: 10.1016/S0967-0661(02)00186-7.
2. R. Soloperto, M. A. Müller, S. Trimpe, and F. Allgöwer, "Learning-based robust model predictive control with state-dependent uncertainty," IFAC-PapersOnLine, vol. 51, no. 20, pp. 442–447, 2018, doi: 10.1016/j.ifacol.2018.11.052.
3. P. Quintanilla, S. J. Neethling, D. Navia, and P. R. Brito-Parada, "A dynamic flotation model for predictive control incorporating froth physics. Part I: Model development," Minerals Engineering, vol. 160, Article 106699, 2021, doi: 10.1016/j.mineng.2021.106699.
4. A. Mesbah, "Stochastic model predictive control: An overview and perspectives for future research," IEEE Control Syst. Mag., vol. 36, no. 6, pp. 30–44, 2016, doi: 10.1109/MCS.2016.2602087.
5. L. Hewing, K. P. Wabersich, M. Menner, and M. N. Zeilinger, "Learning-based model predictive control: Toward safe learning in control," Annu. Rev. Control, Robotics, Auton. Syst., vol. 3, pp. 269–296, 2020, doi: 10.1146/annurev-control-090419-075625.
6. A. D. Bonzanini, J. A. Paulson, and A. Mesbah, "Fast approximate learning-based multistage nonlinear model predictive control using Gaussian processes and deep neural networks," Comput. Chem. Eng., vol. 145, p. 107174, 2021, doi: 10.1016/j.compchemeng.2020.107174.
7. B. Karg and S. Lucia, "Efficient representation and approximation of model predictive control laws via deep learning," IEEE Trans. Cybern., vol. 50, no. 9, pp. 3866–3878, 2020, doi: 10.1109/TCYB.2020.2999556.
8. J. Kabzan, L. Hewing, A. Liniger, and M. N. Zeilinger, "Learning-based model predictive control for autonomous racing," IEEE Robot. Autom. Lett., vol. 4, no. 4, pp. 3363–3370, 2019, doi: 10.1109/LRA.2019.2926677.

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